

Women in maths: mathematician cards for A Level

A Level Math Revision · A Level Maths (Year 12 and Year 13) · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A calculator is allowed in the main activity.

Starter (5 minutes)

1. Is $2 \times 11 + 1$ prime?
2. Find the area of a sector of radius 4 and angle $\pi/4$.
3. Find the gradient of $y = (x^2 + 4)^{-1}$ at $x = 2$.
4. Write down the exact values of $\tan(\pi/4)$ and $\arctan 1$.

Main activity (35 minutes)

Task A: Card 1: Sophie Germain primes

A prime p is a Sophie Germain prime when $2p + 1$ is also prime.

- a. How many Sophie Germain primes are less than 50?
- b. Prove that if $p > 3$ is a Sophie Germain prime then $p \equiv 5 \pmod{6}$.
- c. Disprove by counter-example: 'if p is prime then $2p + 1$ is prime'. Give the smallest counter-example.

Task B: Card 2: the witch of Agnesi

The curve $y = a^3/(x^2 + a^2)$ is known as the witch of Agnesi, after Maria Gaetana Agnesi. Take $a = 2$: $y = 8/(x^2 + 4)$.

- Find the stationary point and show that it is a maximum.
- Show that $d^2y/dx^2 = 16(3x^2 - 4)/(x^2 + 4)^3$, and find the points of inflection.
- (Year 13) Use the substitution $x = 2\tan \theta$ to show that $\int_0^2 8/(x^2 + 4) dx = \pi$.
- (Year 13) Find $\int_0^{2\sqrt{3}} 8/(x^2 + 4) dx$ exactly.

Task C: Card 3: Nightingale's rose diagram

Florence Nightingale is known for polar area diagrams ('rose diagrams'): 12 months drawn as 12 equal sectors, where the area of each sector shows the number. Use 1 mm^2 per case and these made-up numbers.

- Find the angle of each sector in radians.
- A month with 150 cases is drawn. Find the radius of its sector and its arc length, each to 3 significant figures.
- Another month has 4 times as many cases. By what factor is its radius larger? Why is drawing radius in proportion to the number misleading?

Extension

- Start at 89 and keep applying $p \rightarrow 2p + 1$. How many primes do you get before the chain breaks, and what is the first non-prime?
- Show that the area between the witch $y = 8/(x^2 + 4)$ and the x -axis from $x = -N$ to N is $8 \arctan(N/2)$. What does it approach as $N \rightarrow \infty$?