

Women in maths: mathematician cards for A Level: answers

A Level Math Revision · A Level Maths (Year 12 and Year 13) · Teacher answers and notes

The starter is non-calculator. A calculator is allowed in the main activity.

Teacher notes

- Print the three tasks as cards and rotate groups between them.
- The cards name each mathematician only through the maths named after her or that she is known for: invite students to research one of them for homework from a reliable source.
- Card 1 practises three kinds of proof: exhaustion, deduction and counter-example.

Starter

1. Yes, 23 is prime

- 23 is not divisible by 2 or 3, and $5^2 > 23$.

2. 2π

- $\frac{1}{2} \times 16 \times \pi/4 = 2\pi$

3. $-1/16$

- $dy/dx = -2x(x^2 + 4)^{-2} = -4/64 = -1/16$

4. 1 and $\pi/4$

- $\tan(\pi/4) = 1$, so $\arctan 1 = \pi/4$.

Task A: Card 1: Sophie Germain primes

a. 7: 2, 3, 5, 11, 23, 29, 41

- Check $2p + 1$ for each prime below 50: 5, 7, 11, 23, 47, 59, 83 are prime; the rest are not.

b. Proof

- Any integer is $6k$, $6k + 1$, ..., $6k + 5$. A prime above 3 is not divisible by 2 or 3, so it is $6k + 1$ or $6k + 5$.
- If $p = 6k + 1$, then $2p + 1 = 3(4k + 1)$ is a multiple of 3 greater than 3: not prime.
- So $p = 6k + 5$ (proof by exhaustion of the cases).

c. $p = 7$: $2 \times 7 + 1 = 15 = 3 \times 5$

- 2, 3 and 5 give 5, 7 and 11, which are prime.
- 7 gives 15, which is not.

Task B: Card 2: the witch of Agnesi

a. Maximum at (0, 2)

- $dy/dx = -16x/(x^2 + 4)^2 = 0$ only when $x = 0$, where $y = 2$.
- $dy/dx > 0$ for $x < 0$ and < 0 for $x > 0$, so it is a maximum.

b. $(\pm 2/\sqrt{3}, 3/2)$

- Quotient rule on $-16x/(x^2 + 4)^2$ and simplify.
- Zero when $x^2 = 4/3$, and the sign changes there, so $x = \pm 2/\sqrt{3}$.
- $y = 8/(16/3) = 3/2$

c. π

- $dx = 2\sec^2 \theta \, d\theta$ and $x^2 + 4 = 4\sec^2 \theta$.
- The integral becomes $\int 8 \times 2\sec^2 \theta / (4\sec^2 \theta) \, d\theta = \int 4 \, d\theta$.
- $x = 0 \rightarrow \theta = 0$ and $x = 2 \rightarrow \theta = \pi/4$, so the value is $4 \times \pi/4 = \pi$.

d. $4\pi/3$

- Same substitution: the upper limit becomes $\theta = \arctan \sqrt{3} = \pi/3$.
- $4 \times \pi/3 = 4\pi/3$

Task C: Card 3: Nightingale's rose diagram

a. $\pi/6$

- $2\pi \div 12$

b. **23.9 mm; 12.5 mm**

- $\frac{1}{2}r^2(\pi/6) = 150$ gives $r = \sqrt{(1800/\pi)} = 23.93\dots$
- Arc length $= r\theta = 23.93\dots \times \pi/6 = 12.53\dots$

c. **2; doubling the radius would show 4 times the area**

- Area $\propto r^2$, so the radius scales by $\sqrt{4} = 2$.
- If the radius were drawn in proportion, a number twice as big would look 4 times as big.

Extension

1. **6 primes; 5759 = 13 × 443**

- 89, 179, 359, 719, 1439, 2879 are prime.
- $5759 = 13 \times 443$

2. **4π**

- With $x = 2\tan \theta$ the integral is 4θ between $\pm \arctan(N/2)$: $8 \arctan(N/2)$.
- $\arctan(N/2) \rightarrow \pi/2$, so the area $\rightarrow 4\pi$.

Every question is original and every answer was recomputed by computer before publishing.