

Valentine's Day maths for A Level

A Level Math Revision · A Level Maths (Year 12 and Year 13) · 14 February · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A calculator helps in tasks A and C.

Starter (5 minutes)

1. A curve has $x = \cos t$, $y = \sin t$. Find its Cartesian equation.
2. Write $\cos 2t$ in terms of $\cos t$.
3. $x = t^2$, $y = 2t$. Find dy/dx when $t = 1$.
4. Work out 5C_2 .

Main activity (35 minutes)

Task A: The cardioid

A cardioid (heart-shaped curve) has parametric equations $x = 2\cos t - \cos 2t$, $y = 2\sin t - \sin 2t$, for $0 \leq t < 2\pi$.

- a. Find the points where $t = 0$, $t = \pi/2$ and $t = \pi$.
- b. Find dy/dx in terms of t , and its value when $t = \pi/2$.
- c. Show that dx/dt and dy/dt are both 0 when $t = 0$. What does the curve look like there?
- d. The area enclosed by the cardioid is $|\int_0^{2\pi} y (dx/dt) dt|$. Use your calculator to evaluate it, and show it equals 6π .

Task B: A heart curve

Another heart curve is $x = 16\sin^3 t$, $y = 13\cos t - 5\cos 2t - 2\cos 3t - \cos 4t$, for $0 \leq t < 2\pi$.

- Find the points where $t = 0$ and $t = \pi$.
- Find the point where $t = \pi/2$. Why is this the furthest point to the right?
- Explain why the curve is symmetrical in the y -axis.

Task C: A perfect match?

Five friends each write their name on a card. The cards are shuffled and each friend takes one at random.

- In how many ways can the cards be handed out?
- There are 44 ways in which nobody gets their own card. Find the probability that nobody does.
- There are 9 ways for four people to all miss their own cards. Find the probability that exactly one of the five friends gets their own card.
- Find the probability that at least one friend gets their own card, and compare it with $1 - 1/e$.

Extension

- For the cardioid in task A, show that $(dx/dt)^2 + (dy/dt)^2 = 8 - 8\cos t = 16\sin^2(t/2)$. The length of the curve is $\int_0^{2\pi} 4\sin(t/2) dt$: find it.