

Spring egg geometry for A Level

A Level Math Revision · A Level Maths (Year 12 and Year 13) · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A calculator helps in the main activity.

Starter (5 minutes)

1. A curve has $x = 3\cos t$, $y = 2\sin t$. Find its Cartesian equation.
2. Work out $\int_0^{\pi/2} \sin^2 t \, dt$.
3. Find the volume of a sphere of radius 3.
4. For $x = 3\cos t$, $y = 2\sin t$, find dy/dx when $t = \pi/4$.

Main activity (35 minutes)

Task A: An ellipse egg

An egg's outline is $x = 3\cos t$, $y = 2\sin t$, $0 \leq t < 2\pi$ (in cm).

- a. Find the equation of the tangent at $t = \pi/4$, in the form $y = mx + c$.
- b. Find the area of the egg using $\text{area} = 2 \int y \, (dx/dt) \, dt$ over the top half, taking care with the limits.
- c. Find the point in the first quadrant where the gradient is -1 . Give coordinates to 3 significant figures.

Task B: A lopsided egg

A more egg-like outline is $x = 3\cos t$, $y = 2\sin t(1 + 0.1\cos t)$.

- a. Find the points where $t = 0$, $\pi/2$ and π .
- b. Find the height of the top of the egg above the x -axis at $x = 1.5$ and at $x = -1.5$, to 3 significant figures. Which end is fatter?
- c. Show that the area of the lopsided egg is still 6π .

Task C: An inflating egg

A balloon egg keeps its shape as it is blown up: its half-length is always 1.5 times its half-width b , and its volume is $\frac{4}{3}\pi \times 1.5b \times b^2 = 2\pi b^3$.

- a. Find the volume when $b = 2$ cm, exactly and to 3 significant figures.
- b. (Year 13) b increases at 0.1 cm per second. Find the rate of increase of the volume when $b = 2$, to 3 significant figures.
- c. When the volume is 128π cm³, find b and the length of the egg.

Extension

1. The ellipse $x^2/9 + y^2/4 = 1$ is rotated 2π about the x -axis. The volume is $\pi \int_{-3}^3 y^2 dx$. Find it.
2. The same ellipse is rotated about the y -axis instead: volume $\pi \int_{-2}^2 x^2 dy$. Find it.