

Spring egg geometry for A Level: answers

A Level Math Revision · A Level Maths (Year 12 and Year 13) · Teacher answers and notes

The starter is non-calculator. A calculator helps in the main activity.

Teacher notes

- Plot both eggs on graphing software first: students enjoy predicting which has the bigger area before task B.
- In task A (b), the limits matter: as t goes from 0 to π the point moves from right to left, so the integral comes out negative unless the limits are swapped.
- Task C is a short connected-rates-of-change question in a new setting.

Starter

1. $x^2/9 + y^2/4 = 1$

- $\cos t = x/3$, $\sin t = y/2$, and $\cos^2 t + \sin^2 t = 1$.

2. $\pi/4$

- $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$, so the integral is $[t/2 - \sin 2t/4]_0^{\pi/2} = \pi/4$.

3. 36π

- $\frac{4}{3}\pi \times 27$

4. $-2/3$

- $dy/dx = 2\cos t/(-3\sin t) = -(2/3)\cot t = -2/3$ at $t = \pi/4$.

Task A: An ellipse egg

a. $y = -(2/3)x + 2\sqrt{2}$

- The point is $(3/\sqrt{2}, \sqrt{2})$ and the gradient is $-2/3$.
- $y - \sqrt{2} = -(2/3)(x - 3/\sqrt{2})$, so $y = -(2/3)x + \sqrt{2} + \sqrt{2} = -(2/3)x + 2\sqrt{2}$.

b. $6\pi \text{ cm}^2$

- Top half: t from π to 0 as x goes from -3 to 3.
- $2\int_{\pi}^0 2\sin t \times (-3\sin t) dt = 12\int_0^{\pi} \sin^2 t dt = 12 \times \pi/2 = 6\pi$

c. **(2.50, 1.11)**

- $-(2/3)\cot t = -1$ gives $\tan t = 2/3$.
- Then $\cos t = 3/\sqrt{13}$ and $\sin t = 2/\sqrt{13}$, so the point is $(9/\sqrt{13}, 4/\sqrt{13}) = (2.496..., 1.109...)$.

Task B: A lopsided egg

a. **(3, 0), (0, 2), (-3, 0)**

- $t = 0$: (3, 0). $t = \pi/2$: (0, 2). $t = \pi$: (-3, 0).

b. **1.82 and 1.65: the right-hand end is fatter**

- $x = 1.5$ when $t = \pi/3$: $y = 2 \times (\sqrt{3}/2) \times 1.05 = 1.05\sqrt{3} = 1.818...$
- $x = -1.5$ when $t = 2\pi/3$: $y = 2 \times (\sqrt{3}/2) \times 0.95 = 0.95\sqrt{3} = 1.645...$

c. 6π

- Area = $2\int_0^{\pi} 2\sin t(1 + 0.1\cos t) \times 3\sin t dt = 12\int_0^{\pi} \sin^2 t dt + 1.2\int_0^{\pi} \sin^2 t \cos t dt$.
- The second integral is $[\sin^3 t/3]_0^{\pi} = 0$, so the area is $12 \times \pi/2 = 6\pi$.

Task C: An inflating egg

a. **$16\pi \approx 50.3 \text{ cm}^3$**

- $2\pi \times 8 = 16\pi = 50.26\dots$

b. **7.54 cm^3 per second**

- $dV/db = 6\pi b^2 = 24\pi$ at $b = 2$.
- $dV/dt = 24\pi \times 0.1 = 2.4\pi = 7.539\dots$

c. **$b = 4 \text{ cm}$; length 12 cm**

- $2\pi b^3 = 128\pi$ gives $b^3 = 64$, so $b = 4$.
- Length = $2 \times 1.5 \times 4 = 12 \text{ cm}$

Extension

1. **16π**

- $y^2 = 4 - 4x^2/9$
- $\pi[4x - 4x^3/27]_{-3}^3 = \pi(8 + 8) = 16\pi$

2. **24π**

- $x^2 = 9 - 9y^2/4$
- $\pi[9y - 3y^3/4]_{-2}^2 = \pi(12 + 12) = 24\pi$

Every question is original and every answer was recomputed by computer before publishing.