

Pi Day maths for A Level

A Level Math Revision · A Level Maths (Year 12 and Year 13) · 14 March · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A calculator is needed for the main activity.

Starter (5 minutes)

1. A point is chosen at random in a square of side 2. Find the probability that it is inside the circle of radius 1 in the square.
2. Work out $\int_0^\pi \sin x \, dx$.
3. Work out $1 - 1/3 + 1/5 - 1/7$ as a single fraction.
4. Using $\sin \theta \approx \theta$ for small θ (in radians), estimate $12 \sin(\pi/12)$.

Main activity (35 minutes)

Task A: Monte Carlo π and the binomial distribution

A computer throws 1000 random darts at a unit square; each lands inside the quarter circle $x^2 + y^2 \leq 1$ with probability $\pi/4$. Let X be the number of hits, so $X \sim B(1000, \pi/4)$.

- a. Find $E(X)$, to the nearest whole number.
- b. One run gives 791 hits. Find the estimate of π .
- c. Find the standard deviation of X , to 3 significant figures, and so the standard deviation of the estimate $4X/1000$.
- d. Use your calculator's binomial distribution function to find $P(X \geq 800)$, to 3 significant figures. What estimate of π does 800 hits give?

Task B: Buffon's needle

A needle of length l is dropped on a floor with parallel lines d apart, where $l \leq d$. At angle θ to the lines it crosses one with probability $(l \sin \theta)/d$, and θ is equally likely to be anywhere from 0 to π .

- Show that the probability of crossing is $(1/\pi) \int_0^\pi (l \sin \theta)/d \, d\theta = 2l/(\pi d)$.
- Cocktail sticks 4 cm long are dropped on lines 5 cm apart. Find the probability that a stick crosses a line, to 3 significant figures.
- 1000 sticks are dropped and 512 cross a line. Estimate π .

Task C: Archimedes' polygons

A regular n -sided polygon is drawn inside a circle of radius 1. Half its perimeter is $n \sin(\pi/n)$, which approaches π as n grows.

- Find $n \sin(\pi/n)$ for $n = 6, 12$ and 96 , to 4 decimal places.
- Using $\sin \theta \approx \theta - \theta^3/6$, show that $n \sin(\pi/n) \approx \pi - \pi^3/(6n^2)$. Find the smallest n for which this error, $\pi^3/(6n^2)$, is less than 0.001.
- Use the first 5 terms of $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \dots$ to estimate π , to 3 significant figures. Which method is better?

Extension

- Use the substitution $x = \tan \theta$ to show that $\int_0^1 4/(1 + x^2) \, dx = \pi$.
- Use the trapezium rule with 4 strips to estimate $\int_0^1 4/(1 + x^2) \, dx$, to 4 decimal places.