

Pi Day maths for A Level: answers

A Level Math Revision · A Level Maths (Year 12 and Year 13) · 14 March · Teacher answers and notes

The starter is non-calculator. A calculator is needed for the main activity.

Teacher notes

- Pi Day is 14 March because 3/14, written month first, matches $\pi = 3.14\dots$
- Task A (c) explains why random methods are slow: halving the error needs four times as many darts.
- Run task B with real cocktail sticks and a lined floor or sheet of paper; pool the class results.

Starter

1. $\pi/4$

- $\pi \div 4$

2. **2**

- $[-\cos x]_0^\pi = 2$

3. **76/105**

- $(105 - 35 + 21 - 15)/105$

4. π

- $12 \times \pi/12 = \pi$

Task A: Monte Carlo π and the binomial distribution

a. **785**

- $1000 \times \pi/4 = 785.39\dots$

b. **3.164**

- $4 \times 791/1000 = 3.164$

c. **13.0; 0.0519**

- $\sqrt{(1000 \times \pi/4 \times (1 - \pi/4))} = 12.98\dots$
- The estimate is $4X/1000$, so its SD is $0.004 \times 12.98\dots = 0.0519\dots$

d. **0.138; 3.2**

- $P(X \geq 800) = 1 - P(X \leq 799) = 0.1383\dots$
- 800 hits give $4 \times 0.8 = 3.2$.

Task B: Buffon's needle

a. **$2/(\pi d)$**

- $(1/\pi)(1/d) \int_0^\pi \sin \theta \, d\theta = (1/\pi)(1/d) \times 2$

b. **0.509**

- $2 \times 4/(5\pi) = 8/(5\pi) = 0.5092\dots$

c. **3.125**

- $512/1000 \approx 8/(5\pi)$, so $\pi \approx 8000/(5 \times 512) = 3.125$

Task C: Archimedes' polygons

a. **3, 3.1058, 3.1410**

- $6 \sin(\pi/6) = 3$
- $12 \sin(\pi/12) = 3.10582\dots$
- $96 \sin(\pi/96) = 3.14103\dots$

b. **$n = 72$**

- $n(\pi/n - \pi^3/(6n^3)) = \pi - \pi^3/(6n^2)$.
- $\pi^3/(6n^2) < 0.001$ gives $n^2 > 5167.6\dots$, so $n > 71.88\dots$, and $n = 72$.

c. **3.34: the polygons are much better**

- $4 \times (1 - 1/3 + 1/5 - 1/7 + 1/9) = 4 \times 263/315 = 3.339\dots$

Extension

1. **π**

- $dx = \sec^2 \theta d\theta$ and $1 + \tan^2 \theta = \sec^2 \theta$, so the integrand becomes 4.
- θ goes from 0 to $\pi/4$: $4 \times \pi/4 = \pi$.

2. **3.1312**

- $h = 0.25$; the values at 0, 0.25, 0.5, 0.75, 1 are 4, 3.7647..., 3.2, 2.56, 2.
- $0.125 \times (4 + 2 + 2(3.7647\dots + 3.2 + 2.56)) = 3.13117\dots$

Every question is original and every answer was recomputed by computer before publishing.