

Pi Approximation Day puzzles for A Level

A Level Math Revision · A Level Maths (Year 12 and Year 13) · 22 July · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A calculator is useful in the main activity.

Starter (5 minutes)

1. Write $22/7 - 3$ as a fraction.
2. Work out $3 + 1/(7 + 1/15)$ as a single fraction.
3. Convert 180° to radians and $\pi/6$ radians to degrees.
4. Using $\sin \theta \approx \theta$, estimate $\sin 0.02$.

Main activity (30 minutes)

Task A: How good is 22/7?

Use your calculator's value of π .

- a. Find the percentage error when $22/7$ is used for π , to 3 significant figures.
- b. A satellite moves in a circle of radius 7000 km. By how many kilometres is one orbit overestimated if $22/7$ is used for π ? Give 3 significant figures.
- c. Which is the closer approximation to π : $22/7$ or $355/113$? By what factor, to the nearest whole number?

Task B: Continued fractions as a recurrence

$\pi = 3 + 1/(7 + 1/(15 + 1/(1 + 1/(292 + \dots))))$. The approximations p_n/q_n follow $p_n = a_n p_{n-1} + p_{n-2}$ and $q_n = a_n q_{n-1} + q_{n-2}$, with $a = 3, 7, 15, 1, 292, \dots$

- a. Starting from $3/1$ and $22/7$, use the recurrence with $a = 15$ to find the next fraction.
- b. Use $a = 1$ to find the next fraction after that.
- c. Use $a = 292$ to find the next fraction.

Task C: Small angles and π

Radians and small-angle approximations.

- a. Using $\sin \theta \approx \theta$, estimate $180 \sin(1^\circ)$ and compare it with π .
- b. Using $\cos \theta \approx 1 - \theta^2/2$, estimate $\cos 0.1$ and find the error in the approximation, to 2 significant figures.
- c. A wheel turns through 0.4 radians. Using $\pi = 22/7$, how many degrees is this, to the nearest degree?

Extension

1. Archimedes showed that $223/71 < \pi < 22/7$. Find the width of the interval as a single fraction, and the midpoint to 5 decimal places.