

Pi Approximation Day puzzles for A Level: answers

A Level Math Revision · A Level Maths (Year 12 and Year 13) · 22 July · Teacher answers and notes

The starter is non-calculator. A calculator is useful in the main activity.

Teacher notes

- Pi Approximation Day is 22 July because $22/7$, written day first, is the famous fraction for π .
- This is a short set for the last days of term or summer school: each puzzle stands alone.
- Task B uses the same kind of recurrence as the Fibonacci sequence: a nice link back to sequences.

Starter

1. **$1/7$**
 - $22/7 - 21/7 = 1/7$
2. **$333/106$**
 - $7 + 1/15 = 106/15$, so $3 + 15/106 = 333/106$.
3. **π ; 30°**
 - $180^\circ = \pi$ radians, so $\pi/6 = 30^\circ$.
4. **0.02**
 - For small θ in radians, $\sin \theta \approx \theta$.

Task A: How good is $22/7$?

- a. **0.0402%**
 - $(22/7 - \pi) \div \pi \times 100 = 0.04024\dots$
- b. **17.7 km**
 - $2 \times 7000 \times (22/7 - \pi) = 17.70\dots$
- c. **$355/113$, about 4740 times closer**
 - $|22/7 - \pi| = 0.0012644\dots$, $|355/113 - \pi| = 0.00000026676\dots$
 - $0.0012644\dots \div 0.00000026676\dots = 4740.1\dots$

Task B: Continued fractions as a recurrence

- a. **$333/106$**
 - $p = 15 \times 22 + 3 = 333$, $q = 15 \times 7 + 1 = 106$
- b. **$355/113$**
 - $p = 1 \times 333 + 22 = 355$, $q = 1 \times 106 + 7 = 113$
- c. **$103993/33102$**
 - $p = 292 \times 355 + 333 = 103\,993$, $q = 292 \times 113 + 106 = 33\,102$

Task C: Small angles and π

- a. **π ; $180 \sin 1^\circ = 3.1414$ (4 d.p.)**
 - $1^\circ = \pi/180$ radians, so $180 \sin(1^\circ) \approx 180 \times \pi/180 = \pi$.
 - The calculator gives $180 \sin 1^\circ = 3.14143\dots$

b. **0.995; error 0.0000042 (4.2×10^{-6})**

- $1 - 0.01/2 = 0.995$
- $\cos 0.1 = 0.995004165\dots$, so the error is 0.0000041652...

c. **23°**

- $0.4 \times 180/\pi = 0.4 \times 180 \times 7/22 = 22.9\dots$, so 23° .

Extension

1. **$1/497$; 3.14185**

- $22/7 - 223/71 = (1562 - 1561)/497 = 1/497$
- Midpoint = $(22/7 + 223/71) \div 2 = 3.141851\dots$

Every question is original and every answer was recomputed by computer before publishing.