

Halloween maths activities for A Level: answers

A Level Math Revision · A Level Maths (Year 12 and Year 13) · 31 October · Teacher answers and notes

The starter is non-calculator. A calculator is needed for the main activity.

Teacher notes

- Year 12 classes can do the starter, tasks A (a)–(b), C and D, and leave the differential equation and rates of change for a Year 13 lesson.
- In A (c), check that students keep the modulus signs when they integrate and explain why they can drop them here ($0 < Z < 1000$).
- In C (b), the common error is to compare $P(\text{treat} \mid \text{pumpkin})$ with $P(\text{treat})$ using the wrong numbers. Insist on one clear test.

Starter

1. $x = \ln 7$

- Take natural logs of both sides: $x = \ln 7$.

2. $10e^{2x}$

- $d/dx e^{2x} = 2e^{2x}$, so the answer is $10e^{2x}$.

3. 0.1

- $P(A \cap B) = P(A) \times P(B \mid A) = 0.4 \times 0.25 = 0.1$

4. Centre (3, -2), radius 5

- Complete the square: $(x - 3)^2 + (y + 2)^2 = 12 + 9 + 4 = 25$.

Task A: Zombie outbreak

a. $k = \frac{1}{4} \ln 3 \approx 0.275$

- $30e^{4k} = 90$, so $e^{4k} = 3$.
- $4k = \ln 3$, so $k = \frac{1}{4} \ln 3 = 0.2746\dots$

b. 12.8 hours

- $e^{kt} = 100/3$, so $t = \ln(100/3) \div k = 4 \ln(100/3) \div \ln 3 = 12.767\dots$
- 12.8 hours

c. $Z = 1000 \div (1 + 99e^{-0.5t})$

- Separate: $\int 1000 \div (Z(1000 - Z)) dZ = \int 0.5 dt$.
- Partial fractions: $1000 \div (Z(1000 - Z)) = 1/Z + 1/(1000 - Z)$, so $\ln Z - \ln(1000 - Z) = 0.5t + c$.
- At $t = 0$: $c = \ln(10/990) = -\ln 99$. So $Z \div (1000 - Z) = e^{0.5t} \div 99$.
- Rearrange: $Z = 1000 \div (1 + 99e^{-0.5t})$.

d. 13.6 hours; 1000

- $1 + 99e^{-0.5t} = 1000/900 = 10/9$, so $e^{-0.5t} = 1/891$.
- $t = 2 \ln 891 = 13.58\dots$, so 13.6 hours.
- As $t \rightarrow \infty$, $Z \rightarrow 1000$.

Task B: The growing pumpkin (Year 13)

a. **$1/(8\pi) \approx 0.0398$ cm per day**

- $dV/dr = 4\pi r^2 = 400\pi$ at $r = 10$.
- $dr/dt = (dV/dt) \div (dV/dr) = 50 \div 400\pi = 1/(8\pi) = 0.0398\dots$

b. **10 cm^2 per day**

- $S = 4\pi r^2$, so $dS/dr = 8\pi r = 80\pi$.
- $dS/dt = 80\pi \times 1/(8\pi) = 10$

Task C: Trick or treat

a. **0.7 and $27/35 \approx 0.771$**

- $P(\text{treat}) = 0.54 + 0.16 = 0.7$.
- $P(\text{pumpkin} | \text{treat}) = 0.54 \div 0.7 = 27/35$.

b. **No**

- $P(\text{pumpkin}) \times P(\text{treat}) = 0.6 \times 0.7 = 0.42$, but $P(\text{pumpkin} \cap \text{treat}) = 0.54$.
- $0.42 \neq 0.54$, so they are not independent.

c. **0.118**

- $X \sim B(12, 0.7)$. Use the cumulative binomial function.
- $P(X \leq 6) = 0.1178\dots$, so 0.118 .

Task D: The haunted room

a. **Centre $(5, 2)$, radius 5**

- $(x - 5)^2 + (y - 2)^2 = 25 + 4 - 4 = 25$.

b. **$(0, 2)$ and $(5, 7)$; $5\sqrt{2}$**

- Substitute: $x^2 + (x + 2)^2 - 10x - 4(x + 2) + 4 = 0$ gives $2x^2 - 10x = 0$.
- $x = 0$ or 5 , so $(0, 2)$ and $(5, 7)$.
- Length $= \sqrt{5^2 + 5^2} = 5\sqrt{2}$

c. **$y = 7$**

- The radius to $(5, 7)$ goes from $(5, 2)$ straight up, so it is vertical.
- The tangent is perpendicular to it, so it is horizontal: $y = 7$.

Extension

1. **125 zombies per hour, at $t = 2 \ln 99 \approx 9.19$ hours**

- $dZ/dt = 0.5Z - 0.0005Z^2$ is a quadratic in Z with its maximum at $Z = 0.5 \div 0.001 = 500$.
- Greatest rate $= 0.5 \times 500 \times 0.5 = 125$.
- $1000 \div (1 + 99e^{-0.5t}) = 500$ gives $t = 2 \ln 99 = 9.19$ hours.

Every question is original and every answer was recomputed by computer before publishing.