

Fibonacci Day maths for A Level

A Level Math Revision · A Level Maths (Year 12 and Year 13) · 23 November · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A calculator helps in tasks A and C.

Starter (5 minutes)

1. A sequence has $u_1 = 2$, $u_2 = 1$ and $u_{n+2} = u_{n+1} + u_n$. Find u_7 .
2. Solve $x^2 - x - 1 = 0$ exactly.
3. Find $\sum_{r=1}^8 F_r$.
4. $\phi = (1 + \sqrt{5})/2$. Rationalise the denominator to show that $1/\phi = \phi - 1$.

Main activity (35 minutes)

Task A: The ratio limit

Let $r_n = F_{n+1} \div F_n$.

- a. Find r_1 to r_6 .
- b. Show that $r_{n+1} = 1 + 1/r_n$, and use it to find r_7 from r_6 .
- c. Assuming the sequence converges to L , show that $L^2 - L - 1 = 0$. Find L to 3 decimal places and explain why you reject the other root.
- d. The terms lie alternately above and below L . Find r_{10} and how far it is from L , to 2 significant figures.

Task B: Periodic and patterned sequences

Recurrence relations can repeat.

- A sequence has $u_1 = 2$ and $u_{n+1} = 1/(1 - u_n)$. Find u_2 , u_3 and u_4 . What is the period?
- Find $\sum_{r=1}^{30} u_r$ for the sequence in part (a).
- The Fibonacci numbers follow the pattern odd, odd, even, odd, odd, even, ... How many of $F_1, \dots, F_{100}$ are odd?

Task C: The golden ratio in a pentagon

A regular pentagon has sides of length 1.

- Show that each interior angle is 108° .
- Use the cosine rule to find the length of a diagonal, to 4 significant figures.
- Explain why the diagonal is also $2 \cos 36^\circ$, and given that $\cos 36^\circ = (1 + \sqrt{5})/4$, show that the diagonal is exactly ϕ .

Extension

- Show that $u_n = A\phi^n + B\psi^n$ satisfies $u_{n+2} = u_{n+1} + u_n$ for any constants A and B . Then use $F_1 = F_2 = 1$ to find A and B .
- Explain why F_n is the whole number nearest to $\phi^n/\sqrt{5}$, and use this to find F_{25} .