

# e Day maths for A Level

A Level Math Revision · A Level Maths (Year 12 and Year 13) · 7 February · Student sheet

Name \_\_\_\_\_

Class \_\_\_\_\_

Date \_\_\_\_\_

The starter is non-calculator. A calculator is needed for the main activity.

## Starter (5 minutes)

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1. Write down  $\ln e^5$ .
2. Solve  $e^{2x} = 9$ .
3. Differentiate  $3e^{x/2}$ .
4. Simplify  $e^{\ln 7 + \ln 2}$ .

## Main activity (35 minutes)

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### Task A: Compound interest

£2000 is invested for 3 years at a nominal rate of 5% a year. Give money to the nearest penny.

- a. Find the value if interest is compounded annually.
- b. Find the value if interest is compounded monthly.
- c. With continuous compounding the value is  $2000e^{0.15}$ . Find it, and the extra gained over monthly compounding.

### Task B: Where does e come from?

e is the limit of  $(1 + 1/n)^n$  as  $n \rightarrow \infty$ .

- a. Show that the first four terms of the binomial expansion of  $(1 + 1/n)^n$  are  $1 + 1 + (n-1)/(2n) + (n-1)(n-2)/(6n^2)$ . Evaluate them for  $n = 10$ .
- b. As  $n \rightarrow \infty$ ,  $(n-1)/(2n) \rightarrow 1/2!$  and  $(n-1)(n-2)/(6n^2) \rightarrow 1/3!$ , and so on. Add  $1 + 1 + 1/2! + \dots + 1/6!$  to estimate e to 4 decimal places.
- c. The gradient of  $y = a^x$  at  $x = 0$  is close to  $(a^h - 1)/h$  for small  $h$ . Using  $h = 0.001$ , estimate it for  $a = 2$  and for  $a = 3$ , to 3 decimal places. What does this tell you about e?

### Task C: Continuous growth

A population of rare birds on an island is modelled by  $P = 40e^{0.3t}$ , where  $t$  is the time in years.

- a. Write down the initial population.
- b. Find when the population reaches 400, to 3 significant figures.
- c. Show that  $dP/dt = 0.3P$ . Find the rate of growth when the population is 400.

### Extension

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1. Find the smallest whole number  $n$  for which  $(1 + 1/n)^n > 2.7$ .
2. Money grows continuously at 4% a year, so its value is  $Ae^{0.04t}$ . How long does it take to double? Give 3 significant figures, and compare with ' $70 \div 4$ '.