

e Day maths for A Level: answers

A Level Math Revision · A Level Maths (Year 12 and Year 13) · 7 February · Teacher answers and notes

The starter is non-calculator. A calculator is needed for the main activity.

Teacher notes

- e Day is 7 February because 2/7, written month first, matches $e = 2.7...$
- Task B (c) is the key idea: e is the base for which the gradient of a^x at $x = 0$ is 1, which is why $d/dx e^x = e^x$.
- In task A, students often divide the rate by 12 but forget to multiply the number of years by 12.

Starter

1. **5**

- $\ln e^k = k$

2. **$x = \ln 3$**

- $2x = \ln 9 = 2 \ln 3$, so $x = \ln 3$.

3. **$(3/2)e^{x/2}$**

- $3 \times \frac{1}{2} e^{x/2}$

4. **14**

- $e^{\ln 7 + \ln 2} = e^{\ln 14} = 14$

Task A: Compound interest

a. **£2315.25**

- $2000 \times 1.05^3 = 2315.25$

b. **£2322.94**

- $2000 \times (1 + 0.05/12)^{36} = 2322.944...$

c. **£2323.67; 73p more**

- $2000e^{0.15} = 2323.668...$, so £2323.67.
- $2323.67 - 2322.94 = 0.73$

Task B: Where does e come from?

a. **2.57**

- ${}^nC_2/n^2 = n(n-1)/(2n^2) = (n-1)/(2n)$, and ${}^nC_3/n^3 = (n-1)(n-2)/(6n^2)$.
- $n = 10$: $1 + 1 + 9/20 + 72/600 = 1 + 1 + 0.45 + 0.12 = 2.57$

b. **2.7181**

- $1 + 1 + 0.5 + 0.16667 + 0.04167 + 0.00833 + 0.00139 = 2.71806$
- 2.7181 (4 d.p.); $e = 2.71828...$

c. **0.693 and 1.099: e lies between 2 and 3**

- $(2^{0.001} - 1)/0.001 = 0.6934...$
- $(3^{0.001} - 1)/0.001 = 1.0992...$
- e is the base whose gradient at $x = 0$ is exactly 1, so $2 < e < 3$.

Task C: Continuous growth

a. **40**

- $P(0) = 40e^0 = 40$

b. **7.68 years**

- $e^{0.3t} = 10$, so $t = \ln 10 \div 0.3 = 7.675\dots$

c. **120 birds per year**

- $dP/dt = 40 \times 0.3e^{0.3t} = 0.3P$.
- When $P = 400$: $0.3 \times 400 = 120$.

Extension

1. **$n = 74$**

- $(1 + 1/73)^{73} = 2.69989\dots$, which is less than 2.7.
- $(1 + 1/74)^{74} = 2.70013\dots$, which is more than 2.7.

2. **17.3 years; $70 \div 4 = 17.5$**

- $e^{0.04t} = 2$, so $t = \ln 2 \div 0.04 = 17.33\dots$
- $\ln 2 = 0.693\dots$, which is why ' $70 \div \text{rate}$ ' is a good rule of thumb.

Every question is original and every answer was recomputed by computer before publishing.