

# Bonfire Night maths activities for A Level: answers

A Level Math Revision · A Level Maths (Year 12 and Year 13) · 5 November · Teacher answers and notes

The starter is non-calculator. A calculator is needed for the main activity. Take  $g = 9.8 \text{ m s}^{-2}$ .

## Teacher notes

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- Enjoy fireworks at an organised display; this pack is about the maths only.
- Keep exact values ( $35 \sin 60^\circ = 35\sqrt{3}/2$ ) until the last line: early rounding changes the third significant figure in A (d).
- Ask the class what the model ignores (air resistance, the rocket's own thrust) and how each would change the answers.

## Starter

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### 1. 210

- $S_{10} = 10/2 \times (2 \times 3 + 9 \times 4) = 5 \times 42 = 210$

### 2. 16

- $a = 8, r = \frac{1}{2}: 8 \div (1 - \frac{1}{2}) = 16$

### 3. $10/7 \text{ s} \approx 1.43 \text{ s}$

- $v = u - gt = 0$  gives  $t = 14 \div 9.8 = 10/7$ .

### 4. $t = 0$ or $4$

- $4.9t(t - 4) = 0$

## Task A: The firework projectile

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### a. 3.09 s; 46.9 m

- Vertical:  $u = 35 \sin 60^\circ = 30.31... \text{ m s}^{-1}$ .
- Time to the top:  $30.31 \div 9.8 = 3.09 \text{ s}$ .
- Height:  $(35 \sin 60^\circ)^2 \div (2 \times 9.8) = 918.75 \div 19.6 = 46.875$ , so 46.9 m.

### b. 54.1 m

- Horizontal speed:  $35 \cos 60^\circ = 17.5 \text{ m s}^{-1}$ .
- $17.5 \times 3.0929... = 54.13 \text{ m}$

### c. 108 m

- Flight time is twice the time to the top: 6.186 s.
- $17.5 \times 6.186 = 108.25...$ , so 108 m.

### d. 3.71 s

- $30.311t - 4.9t^2 = 30$  gives  $4.9t^2 - 30.311t + 30 = 0$ .
- $t = 1.237$  or  $4.949$ .
- Time above 30 m =  $4.949 - 1.237 = 3.71 \text{ s}$

## Task B: The rocket motor (Year 13)

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### a. $t = 4$ ; $48 \text{ m s}^{-1}$

- $a = dv/dt = 24 - 6t = 0$  when  $t = 4$ .
- $v(4) = 96 - 48 = 48$

b. **256 m**

- $\int_0^8 (24t - 3t^2) dt = [12t^2 - t^3]_0^8 = 768 - 512 = 256$

## Task C: Remember, remember: series

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a. **62; 670**

- $u_{20} = 5 + 19 \times 3 = 62$
- $S_{20} = 20/2 \times (5 + 62) = 670$

b. **25 minutes**

- $n/2 \times (10 + 3(n - 1)) \geq 1000$  gives  $3n^2 + 7n - 2000 \geq 0$ .
- $n \geq (-7 + \sqrt{24\,049}) \div 6 = 24.68\dots$
- So 25 minutes ( $S_{24} = 948$  is not enough).

c. **3905**

- $\sum_{r=1}^5 5^r = 5(5^5 - 1) \div 4 = 5 \times 3124 \div 4 = 3905$

d. **9 rounds**

- $5(5^n - 1) \div 4 > 1\,000\,000$  gives  $5^n > 800\,001$ .
- $n > \log 800\,001 \div \log 5 = 8.45\dots$ , so 9 rounds.

## Extension

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1.  **$y = \sqrt{3}x - (2/125)x^2$**

- $x = 17.5t$ , so  $t = x \div 17.5$ .
- $y = 30.31t - 4.9t^2 = (35 \sin 60^\circ \div 17.5)x - 4.9x^2 \div 306.25$ .
- $35 \sin 60^\circ \div 17.5 = \sqrt{3}$  and  $4.9 \div 306.25 = 0.016 = 2/125$ .

2. **6.79 s; 119 m**

- $-20 = 30.311t - 4.9t^2$  gives  $4.9t^2 - 30.311t - 20 = 0$ .
- $t = (30.311 + \sqrt{(918.75 + 392)}) \div 9.8 = 6.787\dots$
- $17.5 \times 6.787 = 118.8$ , so 119 m.

Every question is original and every answer was recomputed by computer before publishing.