

# Ada Lovelace Day maths for A Level

A Level Math Revision · A Level Maths (Year 12 and Year 13) · Student sheet

Name \_\_\_\_\_

Class \_\_\_\_\_

Date \_\_\_\_\_

The starter is non-calculator. A calculator is needed for tasks B and C.

## Starter (5 minutes)

---

1. Find  $\sum_{r=1}^{20} r$ .
2. A sequence has  $u_1 = 2$  and  $u_{n+1} = 3u_n - 2$ . Find  $u_3$ .
3. Find the sum to infinity of the geometric series with first term 4 and common ratio  $\frac{1}{2}$ .
4. Which term of the sequence 1, 4, 7, 10, ... is 100?

## Main activity (35 minutes)

---

### Task A: Trace the algorithm

Step 1: set  $n = 1$  and  $T = 0$ . Step 2: add  $n^3$  to  $T$ . Step 3: if  $n = 6$ , output  $T$  and stop; otherwise add 1 to  $n$  and go back to step 2.

- a. What does the algorithm output?
- b. Show that the output equals  $(1 + 2 + \dots + 6)^2$ .
- c. Using  $\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$ , find the output when ' $n = 6$ ' is changed to ' $n = 30$ '.

### Task B: An algorithm that solves an equation

The equation  $x^3 - 3x - 5 = 0$  has one real root,  $\alpha$ .

- a. Let  $f(x) = x^3 - 3x - 5$ . Work out  $f(2)$  and  $f(3)$ , and explain why  $\alpha$  lies between 2 and 3.
- b. Use the iteration  $x_{n+1} = \sqrt[3]{3x_n + 5}$  with  $x_1 = 2$  to find  $x_2$ ,  $x_3$  and  $x_4$ , each to 4 decimal places.
- c. Find  $\alpha$  to 3 decimal places, and prove your answer is correct to that accuracy.

### Task C: A savings algorithm

A savings account starts with £500. At the end of each year the balance is multiplied by 1.03 and then £200 is added. Let  $B_n$  be the balance after  $n$  years, so  $B_0 = 500$  and  $B_{n+1} = 1.03B_n + 200$ .

- Find  $B_1$  and  $B_2$ .
- Use a geometric series to show that  $B_n = 500 \times 1.03^n + 200(1.03^n - 1) \div 0.03$ , and find  $B_{10}$  to the nearest penny.
- After how many whole years is the balance first more than £5000?

### Extension

---

- The Bernoulli numbers can be defined by  $B_0 = 1$  and, for  $m \geq 1$ ,  $\sum_{k=0}^m {}^m C_k B_k = 0$ . Find  $B_1$ ,  $B_2$ ,  $B_3$  and  $B_4$ .
- The Bernoulli numbers give formulas for sums of powers, for example  $1^4 + 2^4 + \dots + n^4 = n^5/5 + n^4/2 + n^3/3 - n/30$ . Check this formula when  $n = 3$ , then use it to find  $1^4 + 2^4 + \dots + 10^4$ .