

Ada Lovelace Day maths for A Level: answers

A Level Math Revision · A Level Maths (Year 12 and Year 13) · Teacher answers and notes

The starter is non-calculator. A calculator is needed for tasks B and C.

Teacher notes

- Task B is a good place to show a cobweb diagram on graphing software: the iteration converges because the gradient of $\sqrt[3]{(3x + 5)}$ near the root is less than 1.
- In task C, students who use B_1 as the starting balance get every later answer one year out: check the indexing first.
- Ada Lovelace is widely known for her notes on the Analytical Engine; one of them sets out a method for the Bernoulli numbers in the extension. Keep the lesson about the maths.

Starter

1. **210**

- $20 \times 21 \div 2 = 210$

2. **10**

- $u_2 = 6 - 2 = 4$, $u_3 = 12 - 2 = 10$

3. **8**

- $4 \div (1 - \frac{1}{2}) = 8$

4. **The 34th**

- $u_n = 3n - 2 = 100$ gives $n = 34$.

Task A: Trace the algorithm

a. **441**

- $1 + 8 + 27 + 64 + 125 + 216 = 441$

b. **$21^2 = 441$**

- $1 + 2 + \dots + 6 = 21$ and $21^2 = 441$, the same as the output.

c. **216 225**

- $\frac{1}{4} \times 30^2 \times 31^2 = \frac{1}{4} \times 900 \times 961 = 216\,225$

Task B: An algorithm that solves an equation

a. **$f(2) = -3$, $f(3) = 13$: a change of sign**

- $f(2) = 8 - 6 - 5 = -3$ and $f(3) = 27 - 9 - 5 = 13$.
- f is continuous and changes sign, so there is a root between 2 and 3.

b. **2.2240, 2.2684, 2.2770**

- $x_2 = \sqrt[3]{11} = 2.22398\dots$
- $x_3 = \sqrt[3]{3 \times 2.22398\dots + 5} = 2.26837\dots$
- $x_4 = \sqrt[3]{3 \times 2.26837\dots + 5} = 2.27696\dots$

c. **$\alpha = 2.279$**

- Carrying on the iteration gives 2.2790...
- $f(2.2785) = -0.0065\dots < 0$ and $f(2.2795) = 0.0061\dots > 0$.
- The sign changes, so $2.2785 < \alpha < 2.2795$ and $\alpha = 2.279$ to 3 d.p.

Task C: A savings algorithm

a. **£715 and £936.45**

- $1.03 \times 500 + 200 = 715$
- $1.03 \times 715 + 200 = 936.45$

b. **£2964.73**

- The first £500 has grown to 500×1.03^n . The £200 payments have grown to $200(1 + 1.03 + \dots + 1.03^{n-1}) = 200(1.03^n - 1) \div 0.03$.
- $B_{10} = 500 \times 1.03^{10} + 200(1.03^{10} - 1) \div 0.03 = 2964.734\dots$

c. **17 years**

- Solve $500 \times 1.03^n + 200(1.03^n - 1) \div 0.03 > 5000$, which gives $1.03^n > 11666.67 \div 7166.67 = 1.6279\dots$
- $n > \ln 1.6279 \div \ln 1.03 = 16.49\dots$, so 17 years ($B_{16} = 4882.27$, $B_{17} = 5178.74$).

Extension

1. **$-1/2$, $1/6$, 0 , $-1/30$**

- $m = 1$: $1 + 2B_1 = 0$, so $B_1 = -1/2$.
- $m = 2$: $1 + 3(-1/2) + 3B_2 = 0$, so $B_2 = 1/6$.
- $m = 3$: $1 - 2 + 1 + 4B_3 = 0$, so $B_3 = 0$.
- $m = 4$: $1 - 5/2 + 10/6 + 5B_4 = 0$, so $B_4 = -1/30$.

2. **98 when $n = 3$; 25 333**

- $n = 3$: $243/5 + 81/2 + 9 - 1/10 = 48.6 + 40.5 + 9 - 0.1 = 98$, and $1 + 16 + 81 = 98$.
- $n = 10$: $20\,000 + 5000 + 1000/3 - 1/3 = 25\,333$

Every question is original and every answer was recomputed by computer before publishing.