

Pure Mathematics 4 (P4)

Edexcel IAL Maths · Knowledge organiser

KEY DEFINITIONS

Partial fractions

Splitting a fraction such as $1/((x-1)(x+2))$ into $A/(x-1) + B/(x+2)$.

Implicit differentiation

Differentiating each term with respect to x ; a term in y picks up a factor dy/dx .

Parametric equations

x and y written in terms of t ; $dy/dx = (dy/dt)/(dx/dt)$.

Vector equation of a line

$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ through the point with position vector $\mathbf{a}$, in direction $\mathbf{b}$.

YOU SHOULD BE ABLE TO...

- Write proofs by contradiction, stating the opposite assumption clearly and showing how it leads to an impossible result, such as there being no largest even number.
- Model paths such as a ball's flight or a moving point with parametric equations, and interpret the parameter and the answers in context.
- Use the chain rule to connect rates of change, such as finding how fast the volume of a balloon changes as its radius grows.
- Choose an efficient integration method, from recognition, substitution, parts, partial fractions and trigonometric identities, for a mixed set of integrals.
- Represent vectors in three dimensions with $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, find their magnitudes and unit vectors, and find angles between a vector and the axes.
- Solve multi-step vector geometry problems in 3D, such as finding the area of a triangle or a reflection of a point in a line.

COMMON MISTAKES

- ✗ Differential equations: the constant added too late
- ✗ Binomial (any index): factor not raised to the power
- ✗ Assuming the statement to be proved rather than its negation
- ✗ $A/(x+1)^2$ given without the $A/(x+1)$ term

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Binomial, $|x| < 1$, $n \in \mathbb{Q}$ **B** $(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \dots$

Partial fractions $\frac{\dots}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$,
 $\frac{\dots}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$

Parametric; implicit $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$; $\frac{d}{dx}y^2 = 2y \frac{dy}{dx}$

Integrals **B** $\int \sec^2 kx \, dx = \frac{1}{k} \tan kx$, $\int \tan x \, dx = \ln |\sec x|$,
 $\int \cot x \, dx = \ln |\sin x|$

More integrals **B** $\int \operatorname{cosec} x \, dx = -\ln |\operatorname{cosec} x + \cot x|$,
 $\int \sec x \, dx = \ln |\sec x + \tan x|$

By parts **B** $\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$

Reverse chain $\int \frac{f'(x)}{f(x)} \, dx = \ln |f(x)| + c$

Volume of revolution $V = \pi \int_a^b y^2 \, dx$

Separable DE $\int \frac{dy}{g(y)} = \int f(x) \, dx$

Vectors $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}| \cos \theta$

Line; perpendicular $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$; $\mathbf{a} \cdot \mathbf{b} = 0$

Magnitude $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

WORKED EXAMPLE

Find the exact value of $\int_0^1 x e^{(2x)} \, dx$.

- By parts: $u = x$, $dv/dx = e^{(2x)}$, so $v = \frac{1}{2}e^{(2x)}$
- $\int x e^{(2x)} \, dx = \frac{1}{2}x e^{(2x)} - \frac{1}{4}e^{(2x)}$
- $[\frac{1}{2}x e^{(2x)} - \frac{1}{4}e^{(2x)}]_0^1 = (\frac{1}{2}e^2 - \frac{1}{4}e^2) - (-\frac{1}{4})$

Answer: $(e^2 + 1)/4$