

Pure Mathematics 3 (P3)

Edexcel IAL Maths · Knowledge organiser

KEY DEFINITIONS

Composite function

$fg(x) = f(g(x))$: apply g first.

Modulus function

$|x|$ is the size of x without its sign; the graph of $y = |f(x)|$ reflects the negative parts up.

R form

$a \sin \theta + b \cos \theta = R \sin(\theta + \alpha)$ with $R = \sqrt{a^2 + b^2}$.

Quotient rule

$d/dx(u/v) = (v \, du/dx - u \, dv/dx)/v^2$.

YOU SHOULD BE ABLE TO...

- Simplify rational expressions by factorising and cancelling, and multiply, divide, add and subtract algebraic fractions such as $2/(x+1) - 3/(x^2-1)$.
- Define $\sec$, cosec and $\cot$ as reciprocals of $\cos$, $\sin$ and $\tan$, and find exact values and simplify expressions involving them.
- Write a $\sin x \pm b \cos x$ in the forms $R \sin(x \pm \alpha)$ and $R \cos(x \pm \alpha)$, finding R and α exactly or to a given accuracy.
- Use logarithms to turn $y = ax^n$ and $y = kb^x$ into straight-line graphs, and find the constants from the gradient and intercept.
- Use the chain, product and quotient rules to find tangents, normals and stationary points of curves that combine exponential, logarithmic and trigonometric functions.
- Draw staircase and cobweb diagrams to show whether an iteration converges, and explain why some rearrangements of an equation fail to converge.

COMMON MISTAKES

- ✗ Functions: the domain of an inverse (again), fg versus gf , range
- ✗ Modulus equations and inequalities: extra solutions
- ✗ Cancelling terms rather than factors
- ✗ $Fg(x)$ applied as f first

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Exponential form $e^{x \ln a} = a^x$

B

Reciprocal identities $\sec^2 \theta = 1 + \tan^2 \theta$, $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$

Compound sin **B** $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

Compound cos **B** $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

Compound tan **B** $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double angle $\sin 2A = 2 \sin A \cos A$,
 $\cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$,
 $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

R form $a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha)$,
 $R = \sqrt{a^2 + b^2}$, $\tan \alpha = \frac{b}{a}$

Standard derivatives $(e^{kx})' = ke^{kx}$, $(\ln x)' = \frac{1}{x}$,
 $(\sin kx)' = k \cos kx$, $(\cos kx)' = -k \sin kx$

Trig derivatives **B** $(\tan kx)' = k \sec^2 kx$, $(\sec x)' = \sec x \tan x$,
 $(\cot x)' = -\operatorname{cosec}^2 x$,
 $(\operatorname{cosec} x)' = -\operatorname{cosec} x \cot x$

Product; chain $(uv)' = uv' + vu'$; $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Quotient **B** $\frac{d}{dx} \left(\frac{f}{g} \right) = \frac{f'g - fg'}{g^2}$

Inverse $\frac{dx}{dy} = \frac{1}{dy/dx}$

Integrals $\int e^{kx} dx = \frac{1}{k} e^{kx} + c$, $\int \frac{1}{x} dx = \ln |x| + c$,
 $\int \cos kx dx = \frac{1}{k} \sin kx + c$

Iteration; sign change $x_{n+1} = g(x_n)$; $f(a)f(b) < 0 \Rightarrow$ root in (a, b)

Functions: $fg(x) = f(g(x))$; f^{-1} reflects f in $y = x$; modulus graphs

WORKED EXAMPLE

Write $3 \sin x + 4 \cos x$ in the form $R \sin(x + \alpha)$, with $R > 0$ and $0 < \alpha < \pi/2$.

1. $R = \sqrt{3^2 + 4^2} = 5$

2. $R \cos \alpha = 3$ and $R \sin \alpha = 4$, so $\tan \alpha = 4/3$

Answer: $5 \sin(x + 0.927)$ (α in radians, 3 s.f.)