

Trigonometry

A Level Maths · Knowledge organiser

KEY DEFINITIONS

Radian

The angle that cuts off an arc equal to the radius; π radians = 180° .

Identity

An equation true for every value of the variable, such as $\sin^2\theta + \cos^2\theta = 1$.

Principal value

The single value your calculator gives for an inverse trig function; find the others from the graph.

Harmonic form

$a \sin \theta + b \cos \theta$ written as $R \sin(\theta + \alpha)$, to find maximum and minimum values.

YOU SHOULD BE ABLE TO...

- Use the cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ to find a missing side or a missing angle in a triangle that is not right-angled.
- Sketch transformations of trigonometric graphs such as $y = 2 \sin x$, $y = \cos(x + 30^\circ)$ and $y = \tan 2x$, and describe how the period and amplitude change.
- Solve equations that are quadratic in $\sin$ or $\cos$, such as $2\sin^2x - \sin x - 1 = 0$, using identities and factorising to find every solution.
- Define secant, cosecant and cotangent as reciprocals of cosine, sine and tangent, and find their exact values for standard angles.
- Apply the angle addition formulae to prove identities and to solve equations such as $\sin(x + 30^\circ) = 2 \cos x$ in a given interval.
- Model periodic situations such as tides and Ferris wheels with trigonometric functions, finding greatest and least values and the times they occur.

COMMON MISTAKES

- ✗ $R \cos(\theta - \alpha)$: wrong ratio for $\tan \alpha$ and calculator in the wrong mode
- ✗ Small-angle approximations: missing brackets
- ✗ $(2x)^3$ written as $2x^3$
- ✗ Wrong nCr index

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Sine rule; cosine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}, \quad a^2 = b^2 + c^2 - 2bc \cos A$$

Area; arc; sector (radians)

$$\frac{1}{2}ab \sin C; \quad s = r\theta; \quad A = \frac{1}{2}r^2\theta$$

Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sin^2 \theta + \cos^2 \theta = 1$$

Reciprocal

$$\sec^2 \theta = 1 + \tan^2 \theta, \quad \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$$

Double angle

$$\sin 2A = 2 \sin A \cos A, \\ \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

Double angle (tan)

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Compound sin **B**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

Compound cos **B**

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Compound tan **B**

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Harmonic form, $R > 0$

$$a \sin \theta + b \cos \theta = R \sin(\theta + \alpha), \quad R = \sqrt{a^2 + b^2}, \\ \tan \alpha = \frac{b}{a}$$

Small angles (radians) **B**

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{\theta^2}{2}, \quad \tan \theta \approx \theta$$

Exact values

$$\sin 30^\circ = \frac{1}{2}, \quad \sin 45^\circ = \frac{\sqrt{2}}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}, \\ \tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \tan 60^\circ = \sqrt{3}$$

Inverse trig ranges

$$\arcsin \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \quad \arccos \in [0, \pi], \\ \arctan \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Radians

$$\pi \text{ rad} = 180^\circ$$

WORKED EXAMPLE

Solve $2\sin^2x = \sin x$ for $0^\circ \leq x \leq 360^\circ$.

- $2\sin^2x - \sin x = 0$, so $\sin x(2 \sin x - 1) = 0$
- $\sin x = 0$ gives $x = 0^\circ, 180^\circ, 360^\circ$
- $\sin x = \frac{1}{2}$ gives $x = 30^\circ, 150^\circ$

Answer: $x = 0^\circ, 30^\circ, 150^\circ, 180^\circ, 360^\circ$