

Binomial and normal distributions

A Level Maths · Knowledge organiser

KEY DEFINITIONS

Binomial distribution

$B(n, p)$: the number of successes in n independent trials, each with the same probability p .

Normal distribution

$N(\mu, \sigma^2)$: symmetric and bell-shaped, centred on μ .

Standardising

$z = (x - \mu)/\sigma$ turns any normal variable into $Z \sim N(0, 1)$.

Continuity correction

Adjusting by 0.5 when a continuous normal distribution approximates a discrete one.

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Binomial

$X \sim B(n, p)$ **B**

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Binomial mean,

variance **B**

$$E(X) = np, \quad \text{Var}(X) = np(1-p)$$

Standard normal

$$Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$$

Normal: points of inflection at $\mu \pm \sigma$; about 68%, 95%, 99.7% within 1, 2, 3 s.d.

Normal approx. to

$B(n, p)$ (n large, p near $\frac{1}{2}$) $N(np, np(1-p))$

Use a continuity correction, e.g. $P(X \leq 4) \approx P(Y < 4.5)$

YOU SHOULD BE ABLE TO...

- Write probability distributions for discrete random variables as tables or functions and use the total probability of 1 to find unknown probabilities.
- Recognise situations that fit the binomial distribution $B(n, p)$, state its conditions and calculate probabilities such as $P(X = 3)$ using the formula.
- Describe the shape and properties of the normal distribution $N(\mu, \sigma^2)$, including symmetry about the mean and the proportion of data within one, two and three standard deviations.
- Find probabilities for normally distributed variables, such as $P(X < 52)$ and $P(45 < X < 60)$, using a calculator and sketches.
- Use the standard normal distribution $Z = (X - \mu)/\sigma$ to find an unknown mean or standard deviation, or both, from given probabilities.
- Approximate a binomial distribution $B(n, p)$ with a normal distribution when n is large and p is near 0.5, using a continuity correction.

WORKED EXAMPLE

$X \sim N(20, 3^2)$. Find $P(X > 23)$.

1. $z = (23 - 20)/3 = 1$

2. $P(X > 23) = P(Z > 1) = 1 - 0.8413$

Answer: $P(X > 23) = 0.159$ (3 s.f.)

COMMON MISTAKES

- ✗ Binomial probabilities: "at least" and "more than" read wrongly
- ✗ Normal distribution: continuity correction used on a continuous variable
- ✗ $P(X > 3)$ found as $1 - P(X \leq 4)$
- ✗ Conditions for $B(n, p)$ not stated in context