

Sequences and series

A Level Maths · Knowledge organiser

KEY DEFINITIONS

Arithmetic series

The sum of the terms of an arithmetic sequence, which goes up by d each time.

Geometric series

The sum of the terms of a geometric sequence, which is multiplied by r each time.

Sum to infinity

For a geometric series with $|r| < 1$, the sum gets closer to $a/(1 - r)$.

Binomial expansion

Expanding $(a + b)^n$ using the coefficients nC_r .

YOU SHOULD BE ABLE TO...

- Use Pascal's triangle and factorial notation, including ${}^nC_r = n!/(r!(n - r)!)$, to find binomial coefficients and count arrangements in simple cases.
- Use the first few terms of a binomial expansion to estimate values such as 1.02^8 , and explain why the approximation works when x is small.
- Use the n th term and sum formulae for geometric sequences and series, and solve problems for the number of terms with logarithms where needed.
- Generate sequences from recurrence relations such as $u_{n+1} = 2u_n - 1$, and find unknown constants from given terms of the sequence.
- Model real situations such as savings, loans and population growth with arithmetic and geometric series, and comment on how realistic the models are.
- Split an expression into partial fractions and then use binomial expansions of each part to expand it, stating the range of validity of the result.

COMMON MISTAKES

- ✗ Binomial expansion (rational n): factor not raised to the power, validity not stated
- ✗ Proving by checking a few examples
- ✗ Exhaustion missing a case
- ✗ 'let $n = 3$ ' as a general proof

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Arithmetic n th term $u_n = a + (n - 1)d$

Arithmetic series $S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$
B

Geometric n th term $u_n = ar^{n-1}$

Geometric series $S_n = \frac{a(1-r^n)}{1-r}$, $S_\infty = \frac{a}{1-r}$ ($|r| < 1$)
B

Binomial, $n \in \mathbb{N}$ **B** $(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + b^n$

Binomial coefficient **B** $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

Binomial, $|x| < 1$, $n \in \mathbb{Q}$ **B** $(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \dots$

$(a + bx)^n = a^n(1 + \frac{b}{a}x)^n$, valid for $|x| < |\frac{a}{b}|$

Sigma; recurrence $\sum_{r=1}^n r = \frac{1}{2}n(n + 1)$; $u_{n+1} = f(u_n)$

WORKED EXAMPLE

An arithmetic series has first term 7 and common difference 4. Find the sum of the first 20 terms.

1. $S_n = \frac{1}{2}n[2a + (n - 1)d]$

2. $S_{20} = 10 \times (14 + 19 \times 4) = 10 \times 90$

Answer: $S_{20} = 900$