

Proof

A Level Maths · Knowledge organiser



KEY DEFINITIONS

Proof by deduction

Start from known facts and use logical steps to reach the statement.

Proof by exhaustion

Split into a finite number of cases and check every one.

Counter-example

One example that shows a statement is false.

Proof by contradiction

Assume the statement is false and show this leads to something impossible.

KEY FORMULAS

No formulas to learn here: this topic is about methods and clear reasoning.

WORKED EXAMPLE

Prove that $(2n + 1)^2 - (2n - 1)^2$ is a multiple of 8 for every integer n .

1. $(2n + 1)^2 = 4n^2 + 4n + 1$ and $(2n - 1)^2 = 4n^2 - 4n + 1$
2. The difference is $8n$
3. $8n = 8 \times n$ and n is an integer

Answer: $(2n + 1)^2 - (2n - 1)^2 = 8n$, a multiple of 8

YOU SHOULD BE ABLE TO...

- Write clear proofs by deduction, such as showing that the sum of any three consecutive integers is a multiple of three, setting out each logical step.
- Prove statements by exhaustion, checking every possible case, such as showing that the square of any integer ends in 0, 1, 4, 5, 6 or 9.
- Write proofs by contradiction, such as proving that $\sqrt{2}$ is irrational and that there are infinitely many primes, stating the assumption and the contradiction clearly.

COMMON MISTAKES

- ✗ Proof: "squares are always positive" and no concluding statement
- ✗ Checking a few examples and calling it a proof
- ✗ Not writing a conclusion that states what has been proved