

# Numerical methods

A Level Maths · Knowledge organiser

## KEY DEFINITIONS

### Change of sign

If a continuous  $f(x)$  changes sign between  $a$  and  $b$ , there is a root between them.

### Iteration

Using  $x_{n+1} = g(x_n)$  again and again to home in on a root.

### Newton-Raphson

$x_{n+1} = x_n - f(x_n)/f'(x_n)$ ; it fails if  $f'(x_n)$  is zero or close to it.

### Trapezium rule

Estimates an area with trapezia; it overestimates for a convex curve.

## YOU SHOULD BE ABLE TO...

- Locate roots of  $f(x) = 0$  by finding a change of sign over an interval, and explain when this method can fail, such as at asymptotes.
- Use iterations of the form  $x_{n+1} = g(x_n)$  to approximate roots, and draw staircase and cobweb diagrams to show whether an iteration converges.
- Apply the Newton-Raphson method  $x_{n+1} = x_n - f(x_n)/f'(x_n)$  to find roots, and explain why it can fail near a stationary point.
- Use change of sign, iteration and Newton-Raphson to solve equations that arise from models, judging the accuracy of the answers in context.
- Use the trapezium rule to estimate the value of a definite integral, and decide whether the estimate is an overestimate or underestimate from the curve's shape.

## COMMON MISTAKES

- ✗ Change of sign: incomplete conclusion
- ✗ Change of sign across an asymptote
- ✗ Iterating with rounded values
- ✗ Newton-Raphson failing at a stationary point not explained

## KEY FORMULAS

**B** = given in the exam (formula booklet or list); learn the rest.

Trapezium rule,  $\int_a^b y \, dx \approx \frac{1}{2}h\{y_0 + y_n + 2(y_1 + \dots + y_{n-1})\}$   
 $h = \frac{b-a}{n}$  **B**

Newton-Raphson  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$   
**B**

Sign change: continuous  $f$  with  $f(a)f(b) < 0$  has a root in  $(a, b)$ . Iteration  $x_{n+1} = g(x_n)$ : staircase/cobweb

## WORKED EXAMPLE

**$f(x) = x^3 - 2x - 5$ . Starting at  $x_0 = 2$ , use Newton-Raphson twice. Give  $x_2$  to 4 decimal places.**

1.  $f'(x) = 3x^2 - 2$ ;  $f(2) = -1$  and  $f'(2) = 10$ , so  $x_1 = 2 + \frac{0.1}{10} = 2.1$
2.  $f(2.1) = 0.061$  and  $f'(2.1) = 11.23$ , so  $x_2 = 2.1 - \frac{0.061}{11.23}$

**Answer:  $x_1 = 2.1$  and  $x_2 = 2.0946$  (4 d.p.)**