

# Integration

A Level Maths · Knowledge organiser

## KEY DEFINITIONS

### Indefinite integral

The reverse of differentiation; add a constant  $c$ .

### Definite integral

$\int_a^b f(x) dx$  gives the signed area under the curve from  $a$  to  $b$ .

### Integration by substitution

Change the variable to  $u = g(x)$  to make the integral simpler; change the limits too.

### Integration by parts

$\int u dv/dx dx = uv - \int v du/dx dx$ , for products.

## YOU SHOULD BE ABLE TO...

- Integrate  $x^n$  for  $n \neq -1$ , including terms that must first be rewritten as powers of  $x$ , and understand integration as the reverse of differentiation.
- Use definite integration to find the area between a curve and the  $x$ -axis over a given interval, sketching the region first to check it.
- Integrate standard functions including  $x^n$ ,  $e^x$ ,  $1/x$ ,  $\sin x$ ,  $\cos x$  and  $\sec^2 x$ , and use them to evaluate definite integrals.
- Integrate by substitution, choosing a substitution such as  $u = 2x + 1$ , changing the limits for definite integrals and simplifying before integrating.
- Find areas between curves, and areas under curves defined parametrically, using the integration techniques of this unit and sketches to set up the integrals.
- See a definite integral as the limit of a sum of rectangle areas, and practise mixed integration questions that require choosing the right technique.

## COMMON MISTAKES

- ✗ Modelling: model not written out, limitation not in context
- ✗ Integration by parts: sign errors
- ✗ Constraint not used to reduce to one variable
- ✗  $+ c$  omitted

## KEY FORMULAS

**B** = given in the exam (formula booklet or list); learn the rest.

Standard  $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ ,  $\int \frac{1}{x} dx = \ln|x| + c$ ,  
 $\int e^{kx} dx = \frac{1}{k}e^{kx} + c$

Trig  $\int \cos kx dx = \frac{1}{k}\sin kx + c$ ,  
 $\int \sin kx dx = -\frac{1}{k}\cos kx + c$

More trig **B**  $\int \sec^2 kx dx = \frac{1}{k}\tan kx + c$ ,  
 $\int \tan kx dx = \frac{1}{k}\ln|\sec kx| + c$ ,  
 $\int \cot kx dx = \frac{1}{k}\ln|\sin kx| + c$

Reverse chain **B**  $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$ ,  
 $\int f'(x)[f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1} + c$

Linear inside  $\int f(ax + b) dx = \frac{1}{a}F(ax + b) + c$

By parts **B**  $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Substitution:  $u = g(x)$ , replace  $dx$  by  $\frac{du}{g'(x)}$ , change the limits

Area  $\int_a^b y dx$ ; between curves  $\int_a^b (y_{\text{top}} - y_{\text{bottom}}) dx$ ;  
parametric  $\int y \frac{dx}{dt} dt$

Identities to integrate  $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ ,  
 $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Separable DE  $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x) dx$

## WORKED EXAMPLE

**Evaluate  $\int_1^3 (2x + 1/x) dx$ , giving an exact answer.**

1.  $\int (2x + 1/x) dx = x^2 + \ln x$

2.  $[x^2 + \ln x]_1^3 = (9 + \ln 3) - (1 + 0)$

**Answer:  $8 + \ln 3$**