

Hypothesis testing

A Level Maths · Knowledge organiser



KEY DEFINITIONS

Null hypothesis

H_0 : the parameter has the value assumed so far, such as $p = 0.3$.

Alternative hypothesis

H_1 : what you suspect instead, such as $p > 0.3$ (one-tailed) or $p \neq 0.3$ (two-tailed).

Significance level

The probability threshold, such as 5%, below which you reject H_0 .

Critical region

The values of the test statistic that would lead you to reject H_0 .

YOU SHOULD BE ABLE TO...

- Learn the language of hypothesis testing, including null and alternative hypotheses, the test statistic, the significance level and what it means to reject H_0 .
- Find critical values and critical regions for a binomial hypothesis test at a given significance level, and state the actual significance level of the test.
- Carry out one-tailed and two-tailed binomial hypothesis tests, halving the significance level for each tail, and write a conclusion in the context of the problem.
- Carry out hypothesis tests for zero correlation using the PMCC and critical values from the tables, writing conclusions in context.
- Carry out hypothesis tests for the mean of a normal distribution using the distribution of the sample mean, and write conclusions in context.

COMMON MISTAKES

- ✗ Binomial probabilities: "at least" and "more than" read wrongly
- ✗ Comparing $P(X = x)$ with the significance level instead of a tail probability such as $P(X \geq x)$
- ✗ A conclusion with no context, or one that says H_0 is true

KEY FORMULAS

$\mathbf{B}$ = given in the exam (formula booklet or list); learn the rest.

Sample mean $\mathbf{B}$ $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$, $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

Tests: reject H_0 if p -value $< \alpha$; two-tailed: $\alpha/2$ each tail; PMCC test $H_0: \rho = 0$

WORKED EXAMPLE

$X \sim B(20, p)$. Test $H_0: p = 0.3$ against $H_1: p > 0.3$ at the 5% level, when 10 successes are observed.

1. Under H_0 , $P(X \geq 10) = 0.0480$ (3 s.f.)
2. $0.0480 < 0.05$, so the result is significant

Answer: Reject H_0 : there is evidence at the 5% level that $p > 0.3$