

Exponentials and logarithms

A Level Maths · Knowledge organiser



KEY DEFINITIONS

Exponential function

$y = a^x$ with $a > 0$; $y = e^x$ is the one whose gradient equals its value.

Natural logarithm

$\ln x = \log_e x$, the inverse of e^x .

Laws of logarithms

$\log ab = \log a + \log b$, $\log(a/b) = \log a - \log b$, $\log a^k = k \log a$.

Linearising

Taking logs of $y = ax^n$ or $y = kb^x$ gives a straight line you can read a , n , k , b from.

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Definition $a^x = b \iff x = \log_a b$

Laws $\log_a xy = \log_a x + \log_a y$,
 $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^k = k \log_a x$

Change of base $\log_a x = \frac{\log_b x}{\log_b a}$

Exponential form $a^x = e^{x \ln a}$

Inverses $e^{\ln x} = x$, $\ln e^x = x$

Linearising $y = ax^n \Rightarrow \log y = \log a + n \log x$;
 $y = kb^x \Rightarrow \log y = \log k + x \log b$

YOU SHOULD BE ABLE TO...

- Sketch graphs of exponential functions $y = a^x$ for different values of a , describing their intercepts, asymptotes and whether they show growth or decay.
- Sketch $y = e^x$ and its transformations, and use the fact that the gradient of $y = e^{(kx)}$ is $ke^{(kx)}$ to describe rates of change.
- Understand logarithms as the inverse of exponentials, convert between forms such as $2^5 = 32$ and $\log_2 32 = 5$, and evaluate simple logs.
- Use the laws of logarithms for products, quotients and powers to simplify expressions and solve equations such as $\log_3 x + \log_3 (x - 2) = 1$.
- Use natural logarithms to solve equations involving e^x and use log graphs to turn relationships such as $y = ax^n$ and $y = ab^x$ into straight lines.
- Use logarithms to change exponential relationships such as $y = ab^x$ and $y = ax^n$ into linear form, and interpret regression lines of log data.

WORKED EXAMPLE

Solve $3^x = 20$, giving x to 3 significant figures.

1. Take logs: $x \ln 3 = \ln 20$

2. $x = \ln 20 / \ln 3 = 2.7268...$

Answer: $x = 2.73$ (3 s.f.)

COMMON MISTAKES

- ✗ Modelling: model not written out, limitation not in context
- ✗ Gradient of $e^{(kx)}$ given as $e^{(kx)}$
- ✗ Initial value read as the growth rate
- ✗ $\log_a(1) = 1$