

Differentiation

A Level Maths · Knowledge organiser

KEY DEFINITIONS

Derivative

The gradient function dy/dx : the rate of change of y with x .

Product rule

$d(uv)/dx = u dv/dx + v du/dx$.

Chain rule

$dy/dx = dy/du \times du/dx$ for a function of a function.

Point of inflection

A point where the curve changes between concave and convex, so $f''(x)$ changes sign.

YOU SHOULD BE ABLE TO...

- Estimate the gradient of a curve at a point using chords that get closer to the tangent, and see the gradient as a rate of change.
- Use the derivative to find the gradient of a curve at a point and the equations of the tangent and the normal at that point.
- Sketch the graph of the gradient function $y = f'(x)$ from the graph of $y = f(x)$, linking stationary points to the roots of $f'(x)$.
- Differentiate e^x , $e^{(kx)}$, a^x and $\ln x$, and use these derivatives to find gradients, tangents and stationary points of exponential and logarithmic curves.
- Differentiate $\tan x$, $\sec x$, $\operatorname{cosec} x$ and $\cot x$, and combine these results with the chain, product and quotient rules.
- Use the chain rule to connect rates of change, such as finding how fast the radius of a spreading circle grows, and form simple differential equations.

COMMON MISTAKES

- ✗ $h \rightarrow 0$ taken before simplifying
- ✗ $(x + h)^3$ expanded wrongly
- ✗ Differentiating a quotient without splitting
- ✗ Normal gradient not $-1/m$

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

First principles **B** $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Standard $(x^n)' = nx^{n-1}$, $(e^{kx})' = ke^{kx}$, $(\ln x)' = \frac{1}{x}$,
 $(\sin kx)' = k \cos kx$, $(\cos kx)' = -k \sin kx$

Exponential base a $(a^{kx})' = ka^{kx} \ln a$

Trig **B** $(\tan kx)' = k \sec^2 kx$,
 $(\sec kx)' = k \sec kx \tan kx$,
 $(\cot kx)' = -k \operatorname{cosec}^2 kx$,
 $(\operatorname{cosec} kx)' = -k \operatorname{cosec} kx \cot kx$

Product; chain $(uv)' = uv' + vu'$, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Quotient **B** $\frac{d}{dx} \left(\frac{f}{g} \right) = \frac{f'g - fg'}{g^2}$

Inverse; parametric $\frac{dy}{dx} = \frac{1}{dx/dy}$; $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Implicit $\frac{d}{dx} f(y) = f'(y) \frac{dy}{dx}$, $\frac{d}{dx} (xy) = x \frac{dy}{dx} + y$

Stationary: $f' = 0$; max if $f'' < 0$, min if $f'' > 0$. Convex $f'' > 0$, concave $f'' < 0$; inflection: f'' changes sign

Tangent; normal $y - y_1 = f'(x_1)(x - x_1)$; $m_n = -\frac{1}{f'(x_1)}$

WORKED EXAMPLE

Differentiate $y = x^2 e^{(3x)}$.

1. Product rule with $u = x^2$, $v = e^{(3x)}$

2. $dy/dx = 2x e^{(3x)} + x^2 \times 3e^{(3x)}$

Answer: $dy/dx = x e^{(3x)}(2 + 3x)$