

Algebra and functions

A Level Maths · Knowledge organiser

KEY DEFINITIONS

Discriminant

$b^2 - 4ac$ decides how many real roots $ax^2 + bx + c = 0$ has.

Completing the square

Writing $ax^2 + bx + c$ as $a(x + p)^2 + q$ to find the turning point.

Factor theorem

$f(a) = 0$ exactly when $(x - a)$ is a factor of $f(x)$.

Inverse function

f^{-1} reverses f ; it exists only when f is one-to-one, and its graph reflects f in $y = x$.

YOU SHOULD BE ABLE TO...

- Apply the laws of indices to simplify expressions such as $(2x^3)^2 \div 4x$, then expand products of brackets and factorise by taking out common factors.
- Sketch quadratic graphs showing roots, the y-intercept and the turning point, and use function notation $f(x)$ when finding roots and values.
- Represent linear and quadratic inequalities as regions on a graph, using dotted and solid boundary lines, and describe a shaded region with inequalities.
- Simplify algebraic fractions by factorising and cancelling, and divide polynomials such as $x^3 + 2x^2 - 5x - 6$ by $(x + 1)$ using algebraic long division.
- Classify mappings as one-to-one, many-to-one or not functions, and state the domain and range of functions, including those defined piecewise.
- Split an expression into partial fractions and then use binomial expansions of each part to expand it, stating the range of validity of the result.

COMMON MISTAKES

- ✗ Inverse and composite functions: domain left out, fg and gf mixed up
- ✗ Proof: "squares are always positive" and no concluding statement
- ✗ $(ab)^n = ab^n$
- ✗ $A^{(-n)}$ treated as $-a^n$

KEY FORMULAS

Quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant $b^2 - 4ac$: > 0 two real roots, $= 0$ one repeated root, < 0 no real roots

Completing the square $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$

Indices $a^m a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, $a^{-n} = \frac{1}{a^n}$, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Surds $\sqrt{ab} = \sqrt{a}\sqrt{b}$, $\frac{1}{\sqrt{a} + \sqrt{b}} = \frac{\sqrt{a} - \sqrt{b}}{a - b}$

Factor theorem $f(a) = 0 \iff (x - a)$ is a factor

Partial fractions $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$,
 $\frac{A}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$

Modulus; composite; inverse $\backslash(|x| \ fg(x) = f(g(x))); \ ff^{-1}(x) = x$

Transformations: $f(x) + a$ up a ; $f(x + a)$ left a ; $af(x)$ vertical $\times a$; $f(ax)$ horizontal $\times \frac{1}{a}$

WORKED EXAMPLE

Write $x^2 - 6x + 11$ in the form $(x + p)^2 + q$ and state the minimum value.

$$1. x^2 - 6x = (x - 3)^2 - 9$$

$$2. x^2 - 6x + 11 = (x - 3)^2 - 9 + 11$$

Answer: $(x - 3)^2 + 2$, so the minimum value is 2, at $x = 3$