

Edexcel IAL P4 Differential Equations, Integration by Substitution and Proof by Contradiction Practice Exam

Pearson Edexcel International A Level Mathematics · 12 questions · 58 marks

Name _____

Date _____

Score / 58

- Answer every question. Show your working: most marks are for method.
- Calculator allowed.
- Topics: Integration by substitution; Proof by contradiction; Differential equations.
- The mark scheme is a separate free PDF: alevelmathrevision.com/alevel-frontend/downloads/practice-exams/ial/edexcel-ial-p4-differential-equations-integration-by-substitution-and-proof-by-contradiction-practice-exam-mark-scheme.pdf

1. Use the substitution $u = \tan x$ to find the exact value of $\int_0^{\frac{\pi}{4}} \tan^3 x \sec^2 x \, dx$. [3] [3]

2. Use the substitution $u = 1 + \sin x$ to find the exact value of $\int_0^{\frac{\pi}{6}} \frac{\cos x}{(1 + \sin x)^2} \, dx$. [4] [4]

3. Prove by contradiction that there is no greatest even integer. [4]

4. Prove by contradiction that if n is an integer and n^2 is odd, then n is odd. [4]

5. Given that $\sqrt{2}$ is irrational, prove by contradiction that $5 - 3\sqrt{2}$ is irrational. [4]



6. Prove by contradiction that $x^2 - 6x + 10 > 0$ for all real values of x . [4]

7. Solve the differential equation $\frac{dy}{dx} = (y + 1) \cos x$, given that $y = 1$ when $x = 0$. Give y in terms of x , and hence find the exact value of y when $x = \frac{\pi}{2}$. [5]

8. Use the substitution $u = \sqrt{x - 1}$ to find the exact value of $\int_2^5 x\sqrt{x - 1} dx$. [5]

9. Prove by contradiction that there are no positive integers x and y such that $x^2 - y^2 = 1$. [5]

10. Prove by contradiction that $\log_2 3$ is irrational. [5]

11. Use the substitution $u = 1 + x^2$ to show that $\int_0^1 x^3 \sqrt{1 + x^2} dx = \frac{2}{15} (\sqrt{2} + 1)$. [6]



12. The number of fish, P hundred, in a lake t years after it is stocked is modelled by $\frac{dP}{dt} = \frac{P(10-P)}{20}$, with $P = 2$ when $t = 0$. [9]
- (a) Express $\frac{1}{P(10-P)}$ in partial fractions. (2)
- (b) Hence show that $P = \frac{10}{1+4e^{-\frac{t}{2}}}$. (5)
- (c) Find the exact time at which the number of fish reaches 800. (2)

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