

Edexcel IAL P4 Differential Equations, Integration by Substitution and Proof by Contradiction Practice Exam: Mark Scheme

Pearson Edexcel International A Level Mathematics · 12 questions · 58 marks

- M marks are for a correct method; A marks for an accurate answer that follows a correct method; B marks are independent of method.
- Accept any correct method that reaches a correct answer, unless the question says which method to use.
- Practise these topics online with instant feedback: alevelmathrevision.com/alevel-frontend/alevel-unit.html?course=ial&unit=4

Question 1

3 marks

M1 $du = \sec^2 x \, dx$, limits 0 and 1

M1 $\left[\frac{u^4}{4} \right]_0^1$

A1 $\frac{1}{4}$

Final answer: $\frac{1}{4}$

Question 2

4 marks

M1 $du = \cos x \, dx$, $\int u^{-2} \, du$

M1 $-\frac{1}{u}$

M1 limits $u = 1$ and $u = \frac{3}{2}$: $-\frac{2}{3} + 1$

A1 $\frac{1}{3}$

Final answer: $\frac{1}{3}$

Question 3

4 marks

Assume there is a greatest even integer, N . **M1**

Consider $N + 2$: it is an integer and $N + 2 = 2\left(\frac{N}{2} + 1\right)$ is even. **M1**

$N + 2 > N$ **A1**

so $N + 2$ is an even integer greater than N , contradicting the assumption. Hence there is no greatest even integer. **A1**

Question 4

4 marks

Assume n^2 is odd and n is even. **M1**

Then $n = 2k$ for some integer k **M1**

so $n^2 = 4k^2 = 2(2k^2)$, which is even **A1**

This contradicts n^2 being odd, so n must be odd. **A1**



Question 5

4 marks

Assume $5 - 3\sqrt{2} = \frac{p}{q}$, where p, q are integers and $q \neq 0$. **M1**

Then $\sqrt{2} = \frac{5-p}{3q}$ **M1**

$= \frac{5q-p}{3q}$ **A1**

$5q - p$ and $3q$ are integers with $3q \neq 0$, so $\sqrt{2}$ would be rational: a contradiction. Hence $5 - 3\sqrt{2}$ is irrational. **A1**

Question 6

4 marks

Assume there is a real x with $x^2 - 6x + 10 \leq 0$. **M1**

Completing the square: $x^2 - 6x + 10 = (x - 3)^2 + 1$ **M1**

so $(x - 3)^2 + 1 \leq 0$ **A1**

but $(x - 3)^2 \geq 0$, so $(x - 3)^2 + 1 \geq 1$: a contradiction. Hence $x^2 - 6x + 10 > 0$ for all real x . **A1**

Question 7

5 marks

$\int \frac{1}{y+1} dy = \int \cos x dx$ **M1**

$\ln(y+1) = \sin x + c$ **A1**

$x = 0, y = 1: c = \ln 2$ **M1**

$y = 2e^{\sin x} - 1$ **A1**

$x = \frac{\pi}{2}: y = 2e - 1$ **A1**

Question 8

5 marks

B1 $x = u^2 + 1, dx = 2u du$

M1 $\int (u^2 + 1)u \cdot 2u du = \int (2u^4 + 2u^2) du$

M1 $\frac{2u^5}{5} + \frac{2u^3}{3}$

M1 limits $u = 1$ and $u = 2$

A1 $\frac{256}{15}$

Final answer: $\frac{256}{15}$

Question 9

5 marks

Assume there are positive integers x, y with $x^2 - y^2 = 1$. **M1**

Then $(x - y)(x + y) = 1$ **M1**

$x + y$ is a positive integer, so $x + y = 1$ and $x - y = 1$ **A1**

giving $x = 1$ and $y = 0$ **A1**

but $y = 0$ is not positive, a contradiction. So no such positive integers exist. **A1**

Question 10

5 marks

Assume $\log_2 3 = \frac{a}{b}$, where a and b are positive integers ($\log_2 3 > 0$). **M1**

Then $2^{\frac{a}{b}} = 3$ **M1**

so $2^a = 3^b$ **A1**

2^a is even (as $a \geq 1$) but 3^b is odd **M1**

which is a contradiction, so $\log_2 3$ is irrational. **A1**



Question 11

6 marks

B1 limits $u = 1$ and $u = 2$

M1 $x dx = \frac{1}{2} du$ and $x^2 = u - 1$

A1 $\frac{1}{2} \int (u-1)u^{\frac{1}{2}} du = \frac{1}{2} \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$

M1 $\frac{1}{2} \left[\frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}} \right]$

dM1 applies limits: $\frac{1}{2} \left[\left(\frac{8\sqrt{2}}{5} - \frac{4\sqrt{2}}{3} \right) - \left(\frac{2}{5} - \frac{2}{3} \right) \right]$

A1* $\frac{4\sqrt{2}+4}{30} = \frac{2}{15}(\sqrt{2} + 1)$

Final answer: $\frac{2}{15}(\sqrt{2} + 1)$

Question 12

9 marks

a. $1 = A(10 - P) + BP$ **M1**

$A = B = \frac{1}{10} : \frac{1}{10P} + \frac{1}{10(10-P)}$ **A1**

b. $\int \frac{1}{P(10-P)} dP = \int \frac{1}{20} dt$ **M1**

$\frac{1}{10} \ln P - \frac{1}{10} \ln(10 - P) = \frac{t}{20} + c$ **M1 A1**

$t = 0, P = 2: c = \frac{1}{10} \ln \frac{1}{4}, \text{ so } \frac{P}{10-P} = \frac{1}{4} e^{\frac{t}{2}}$ **M1**

$4P = (10 - P)e^{\frac{t}{2}} \Rightarrow P = \frac{10e^{t/2}}{4+e^{t/2}} = \frac{10}{1+4e^{-\frac{t}{2}}}$ * **A1**

c. $8 = \frac{10}{1+4e^{-t/2}} \Rightarrow e^{-\frac{t}{2}} = \frac{1}{16}$ **M1**

$t = 2 \ln 16 = 8 \ln 2 (\approx 5.55 \text{ years})$ **A1**

End of mark scheme · More free exams: [alevelmathrevision.com/alevel-frontend/free-practice-exams](https://www.alevelmathrevision.com/alevel-frontend/free-practice-exams)

