

Edexcel IAL P3 Composite and Inverse Functions and Numerical Methods Practice Exam: Mark Scheme

Pearson Edexcel International A Level Mathematics · 12 questions · 59 marks

- M marks are for a correct method; A marks for an accurate answer that follows a correct method; B marks are independent of method.
- Accept any correct method that reaches a correct answer, unless the question says which method to use.
- Practise these topics online with instant feedback: alevelmathrevision.com/alevel-frontend/alevel-unit.html?course=ial&unit=3

Question 1

2 marks

M1 $f(1.5) = -0.625$, $f(1.6) = 0.296$

A1 sign change and f continuous, so a root in $[1.5, 1.6]$

Final answer: $f(1.5) = -0.625 < 0$, $f(1.6) = 0.296 > 0$: a root lies in $[1.5, 1.6]$

Question 2

2 marks

B1 f is not continuous on $[1, 3]$: it has a vertical asymptote at $x = 2$

B1 the only root of $f(x) = 0$ is $x = -1$, which is not in $[1, 3]$

Final answer: the sign change is caused by the asymptote at $x = 2$; the only root is $x = -1$

Question 3

3 marks

M1 $x_1 = (5.2)^{\frac{1}{5}}$

A1 $x_1 = 1.3906$

A1 $x_2 = 1.3891$, $x_3 = 1.3888$

Final answer: $x_1 = 1.3906$, $x_2 = 1.3891$, $x_3 = 1.3888$

Question 4

4 marks

M1 $fg(x) = |x^2 - 2x - 3|$; $x^2 - 2x - 3 = 5 \Rightarrow x^2 - 2x - 8 = 0$

A1 $x = 4$, $x = -2$

M1 $x^2 - 2x - 3 = -5 \Rightarrow x^2 - 2x + 2 = 0$

A1 discriminant $-4 < 0$: no further solutions

Final answer: $x = -2$, $x = 4$



Question 5**5 marks**

M1 (a) $g(1) = 2$

A1 $fg(1) = 5$

M1 (b) $x^2 = \frac{y+3}{2}$, positive root

A1 $f^{-1}(x) = \sqrt{\frac{x+3}{2}}$

B1 $x \geq -3$

Final answer: (a) 5 (b) $f^{-1}(x) = \sqrt{\frac{x+3}{2}}$, $x \geq -3$ **Question 6****5 marks**

M1 (a) $e^{\ln(x-1)} + 2$

A1 $fg(x) = x + 1$, $x > 1$

B1 (b) $gf(x) = \ln(e^x + 1)$

M1 (c) $e^x + 1 = e^2$

A1 $x = \ln(e^2 - 1)$

Final answer: (a) $x + 1$ (b) $\ln(e^x + 1)$ (c) $x = \ln(e^2 - 1)$ **Question 7****5 marks**

M1 (a) $f(3.3) = 0.0177$, $f(3.4) = -0.0136$

A1 sign change and f continuous, so a root

B1 (b) $\frac{x}{2} = \ln(x + 2) \Rightarrow x = 2 \ln(x + 2)$

M1 (c) $x_1 = 2 \ln 5$

A1 $x_1 = 3.2189$, $x_2 = 3.3046$, $x_3 = 3.3371$

Final answer: (c) $x_1 = 3.2189$, $x_2 = 3.3046$, $x_3 = 3.3371$ **Question 8****6 marks**

M1 (a) $3x - 1 = 3(x + 2) - 7$

A1* $3 - \frac{7}{x+2}$

B1 (b) $f(x) < 3$

M1 (c) $y(x + 2) = 3x - 1 \Rightarrow x(3 - y) = 2y + 1$

A1 $f^{-1}(x) = \frac{2x+1}{3-x}$

B1 $x < 3$

Final answer: (b) $f(x) < 3$ (c) $f^{-1}(x) = \frac{2x+1}{3-x}$, $x < 3$ 

Question 9**6 marks****B1** (a) $f(x) \leq 5$ **M1** $x^2 = 5 - y$, negative root as $x \leq 0$ **A1** $f^{-1}(x) = -\sqrt{5-x}$, $x \leq 5$ **M1** (b) $5 - (2x + 1)^2 = -4 \Rightarrow (2x + 1)^2 = 9$ **M1** requires $g(x) = 2x + 1 \leq 0$ (domain of f), so $2x + 1 = -3$ **A1** $x = -2$ only**Final answer:** (a) $f(x) \leq 5$, $f^{-1}(x) = -\sqrt{5-x}$ (b) $x = -2$ **Question 10****7 marks****M1** (a) $e^{2.5} - 12.5 = -0.318$, $e^{2.6} - 13 = 0.464$ **A1** sign change and continuous, so $\alpha \in [2.5, 2.6]$ **M1** (b) $x_1 = \frac{e^{2.5}}{5}$ **A1** $x_1 = 2.4365$, $x_2 = 2.2866$, $x_3 = 1.9683$ **B1** (c) the iterates move away from α , so this formula does not converge to α **M1** (d) $x_1 = \ln 12.5$ **A1** $x_1 = 2.5257$, $x_2 = 2.5360$, $x_3 = 2.5400$ **Final answer:** (b) 2.4365, 2.2866, 1.9683 (c) moves away from α (d) 2.5257, 2.5360, 2.5400**Question 11****7 marks****M1** (a) $x_2 = \sqrt{5.5}$ **A1** $x_2 = 2.3452$ **A1** $x_3 = 2.2977$, $x_4 = 2.3034$ **M1** (b) $\alpha = \sqrt{4 + \frac{3}{\alpha}} \Rightarrow \alpha^2 = 4 + \frac{3}{\alpha}$ **A1*** multiplies by α : $\alpha^3 - 4\alpha - 3 = 0$ **M1** (c) $(\alpha + 1)(\alpha^2 - \alpha - 3) = 0$ **A1** $\alpha = \frac{1+\sqrt{13}}{2}$ (as $\alpha > 0$)**Final answer:** (a) $x_2 = 2.3452$, $x_3 = 2.2977$, $x_4 = 2.3034$ (c) $\alpha = \frac{1+\sqrt{13}}{2}$ 

Question 12

7 marks

M1 (a) product rule on $x \ln x$: $\ln x + 1$

A1 $\frac{dy}{dx} = \ln x + 1 - \frac{x}{3}$

A1* $\frac{dy}{dx} = 0 \Rightarrow x = 3 + 3 \ln x$

M1 (b) $\frac{dy}{dx}$ at 9.8: 0.0157; at 9.9: -0.0075

A1 sign change and continuous, so $\beta \in [9.8, 9.9]$

M1 (c) $x_1 = 3 + 3 \ln 10$

A1 $x_1 = 9.9078, x_2 = 9.8800$

Final answer: (c) $x_1 = 9.9078, x_2 = 9.8800$

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