

Edexcel IAL P2 Mathematical Proof Practice Exam: Mark Scheme

Pearson Edexcel International A Level Mathematics · 12 questions · 45 marks

- M marks are for a correct method; A marks for an accurate answer that follows a correct method; B marks are independent of method.
- Accept any correct method that reaches a correct answer, unless the question says which method to use.
- Practise these topics online with instant feedback: alevelmathrevision.com/alevel-frontend/alevel-unit.html?course=ial&unit=2

Question 1

2 marks

Substitutes positive integers into $n^2 + 3n + 1$ until a value that is not prime is found, e.g. $n = 6$: $36 + 18 + 1 = 55$
M1

$55 = 5 \times 11$, so 55 is not prime and the statement is false **A1**

Note: $n = 1, \dots, 5$ give 5, 11, 19, 29, 41, which are all prime. Any value shown to be composite earns both marks (e.g. $n = 11$: $155 = 5 \times 31$).

Final answer: $n = 6$: $6^2 + 3 \times 6 + 1 = 55 = 5 \times 11$, which is not prime.

Question 2

2 marks

Chooses a value $x \leq -\frac{1}{2}$ and evaluates both sides, e.g. $x = -1$: $(x + 1)^2 = 0$ and $x^2 = 1$ **M1**

$0 < 1$, so $(x + 1)^2 > x^2$ is false for $x = -1$; the statement is not true **A1**

Note: $(x + 1)^2 - x^2 = 2x + 1$, so every $x \leq -\frac{1}{2}$ is a counter-example.

Final answer: e.g. $x = -1$: $(x + 1)^2 = 0$, $x^2 = 1$, and $0 < 1$.

Question 3

3 marks

Four consecutive integers written as $n, n + 1, n + 2, n + 3$, where n is an integer **B1**

Sum = $4n + 6$ **M1**

$4n + 6 = 4(n + 1) + 2$, which leaves remainder 2 when divided by 4, so the sum is never a multiple of 4 **A1** (cso)

Final answer: $n + (n + 1) + (n + 2) + (n + 3) = 4n + 6 = 4(n + 1) + 2$: always 2 more than a multiple of 4.

Question 4

3 marks

Lists all six two-digit square numbers: 16, 25, 36, 49, 64, 81 **B1**

Subtracts 1 from each and shows the result is not prime, at least three correct, e.g. $16 - 1 = 15 = 3 \times 5$ **M1**

All six: $15 = 3 \times 5$, $24 = 2 \times 12$, $35 = 5 \times 7$, $48 = 2 \times 24$, $63 = 7 \times 9$, $80 = 2 \times 40$; none is prime, so no two-digit square is one more than a prime **A1** (cso)

Note: B0 if a two-digit square is missing. Writing $n^2 - 1 = (n - 1)(n + 1)$, with both factors greater than 1, for each of $n = 4, \dots, 9$ is an acceptable method for M1 A1.

Final answer: The two-digit squares are 16, 25, 36, 49, 64, 81; one less gives 15, 24, 35, 48, 63, 80, none of which is prime.



Question 5

3 marks

Consecutive even numbers written as $2n$ and $2n + 2$, where n is an integer **B1**

$$2n(2n + 2) + 1 = 4n^2 + 4n + 1 \text{ M1}$$

$= (2n + 1)^2$, the square of an integer, so always a square number **A1** (cso)

Final answer: $2n(2n + 2) + 1 = 4n^2 + 4n + 1 = (2n + 1)^2$.

Question 6

3 marks

Starts from a true statement, e.g. $(x - 2y)^2 \geq 0$ for all real x, y **M1**

$$x^2 - 4xy + 4y^2 \geq 0 \Rightarrow x^2 + 4y^2 \geq 4xy \text{ A1 (cso)}$$

Equality when $x = 2y$ **B1**

Note: working backwards from the result earns M1 A0 unless the argument is reversed with a conclusion.

Final answer: $(x - 2y)^2 \geq 0$ gives $x^2 + 4y^2 \geq 4xy$; equality when $x = 2y$.

Question 7

4 marks

(a) Attempts $(2x - 3)^2 \pm \dots$ **M1**

$$(2x - 3)^2 + 2 \text{ (or } (-2x + 3)^2 + 2) \text{ A1}$$

(b) $(2x - 3)^2 \geq 0$ for all real x , so $(2x - 3)^2 + 2 \geq 2 > 0$ **B1**

(c) 2 (when $x = \frac{3}{2}$) **B1ft**

Final answer: (a) $(2x - 3)^2 + 2$ (b) $(2x - 3)^2 + 2 \geq 2 > 0$ (c) 2

Question 8

5 marks

(a) Considers every case: $n = 5k, 5k \pm 1, 5k \pm 2$ (or $5k, 5k + 1, 5k + 2, 5k + 3, 5k + 4$), k an integer **M1**

Squares at least two cases correctly, e.g. $(5k \pm 1)^2 = 25k^2 \pm 10k + 1$ **M1**

All cases correct: $(5k)^2 = 5(5k^2)$; $(5k \pm 1)^2 = 5(5k^2 \pm 2k) + 1$; $(5k \pm 2)^2 = 5(5k^2 \pm 4k) + 4$ **A1**

Conclusion: every integer is in one of the cases, so the remainder is always 0, 1 or 4 **A1** (cso)

(b) A whole number ending in 2 or 7 leaves remainder 2, and one ending in 3 or 8 leaves remainder 3, on division by 5; by (a) a square leaves remainder 0, 1 or 4, so it cannot end in 2, 3, 7 or 8 **B1**

Final answer: (a) Remainders 0, 1, 4 in the cases $5k, 5k \pm 1, 5k \pm 2$. (b) Numbers ending in 2 or 7 leave remainder 2, and those ending in 3 or 8 leave remainder 3, on division by 5; a square leaves only 0, 1 or 4.

Question 9

4 marks

Finds the lengths of the four sides, e.g. $AB = \sqrt{6^2 + 2^2}$ **M1**

$AB = BC = CD = DA = \sqrt{40}$, so $ABCD$ is a rhombus **A1**

Tests for a right angle, e.g. gradient $AB = \frac{1}{3}$, gradient $BC = 3$ **M1**

$\frac{1}{3} \times 3 = 1 \neq -1$, so AB is not perpendicular to BC : not a square **A1** (cso)

Final answer: All sides $\sqrt{40}$; $m_{AB} \times m_{BC} = 1 \neq -1$, so no right angle.



Question 10

5 marks

(a) Starts from a true statement: $(p - 3)^2 \geq 0$ for all real p , so $p^2 - 6p + 9 \geq 0$ **M1**

$p^2 + 9 \geq 6p$, then divides by p , stating that the inequality is unchanged because $p > 0$ **M1**

Hence $p + \frac{9}{p} \geq 6$ **A1** (cso)

Note: an argument that starts from the required result (e.g. multiplies it by p to reach $(p - 3)^2 \geq 0$) earns M1 M1 A0 unless it is reversed with a clear conclusion.

(b) Substitutes a negative value, e.g. $p = -1$ **M1**

$-1 - 9 = -10$, and $-10 < 6$, so the inequality is false **A1**

Final answer: (a) $(p - 3)^2 \geq 0$ with $p > 0$ (b) e.g. $p = -1$ gives $-10 < 6$.

Question 11

5 marks

(a) Tests $n = 3$: $3^3 = 27$ and $3^3 = 27$ **M1**

27 is not greater than 27, so the claim is false **A1**

Note: $n = 1$ and $n = 2$ satisfy the claim ($3 > 1$, $9 > 8$); $n = 3$ is the only counter-example.

(b) Evaluates both sides for $n = 4, 5, 6, 7$ **M1**

$81 > 64$, $243 > 125$, $729 > 216$, $2187 > 343$ **A1**

Conclusion: true for every n in the range **A1**

Final answer: (a) $n = 3$: $27 = 27$ (b) $81 > 64$, $243 > 125$, $729 > 216$, $2187 > 343$

Question 12

6 marks

Factorises: $n^4 - n^2 = n^2(n - 1)(n + 1)$ **M1**

$(n - 1)n(n + 1)$ is a product of three consecutive integers, so one factor is a multiple of 3 **B1**

Considers n even and n odd **M1**

n even: $n = 2k$, so $n^2 = 4k^2$ is a multiple of 4 **A1**

n odd: $n - 1$ and $n + 1$ are both even, so $(n - 1)(n + 1)$ is a multiple of 4 **A1**

The number is a multiple of 3 and of 4, which have no common factor, so it is a multiple of 12 **A1** (cso)

Final answer: $n^2(n - 1)(n + 1)$: a multiple of 3 (three consecutive integers) and of 4 (both parities), hence of 12.

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