FORMULAS YOU MUST MEMORISE

A Level Mathematics (UK)

Not in the Edexcel 9MA0 formulae booklet: learn these by heart. AQA and OCR print slightly different booklets: the full sheet ticks what each prints.

A O = AQA or OCR prints it (Edexcel does not). Everything here is one to learn for Edexcel 9MA0.

Algebra & functions

E A O **Quadratic formula:** $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
When: Solving quadratics that do not factorise; "for what k" root questions

E A O **Discriminant b^2-4ac: >0 two real roots, $=0$ one repeated root, <0 no real roots**
When: Decide how many real roots a quadratic has

E A O **Completing the square:** $ax^2+bx+c = a\left(x+\frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$
When: Completed-square form: solve equations; find the turning point

E A O **Indices:** $a^m a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, $a^{-n} = \frac{1}{a^n}$, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
When: Negative and fractional indices; rewrite terms as ax^n

E A O **Surds:** $\sqrt{ab} = \sqrt{a}\sqrt{b}$, $\frac{1}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{a}-\sqrt{b}}{a-b}$
When: Simplify surds; rationalise denominators with the conjugate

E A O **Factor theorem:** $f(a)=0 \iff (x-a)$ **is a factor**
When: Test whether $(x-a)$ is a factor; fully factorise cubics

E A O **Partial fractions:** $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$, $\frac{\dots}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
When: Proper fractions with distinct or repeated linear factors

E A O **Modulus; composite; inverse:** $|x|<a \iff -a<x<a$; $fg(x) = f(g(x))$; $ff^{-1}(x) = x$
When: Composite and inverse functions with their domains and ranges

E A O **Transformations:** $f(x)+a$ **up** a; $f(x+a)$ **left** a; $af(x)$ **vertical** $\times a$; $f(ax)$ **horizontal** $\times \frac{1}{a}$
When: Combine transformations, tracking key points

Differentiation

E A O **Standard:** $(x^n)' = nx^{n-1}$, $(e^{kx})' = ke^{kx}$, $(\ln x)' = \frac{1}{x}$, $(\sin kx)' = k\cos kx$, $(\cos kx)' = -k\sin kx$
When: Every calculus question with exponentials, logs or trig

E A O **Exponential base a:** $(a^{kx})' = ka^{kx}\ln a$
When: Differentiate a^x

E A O **Product; chain:** $(uv)' = uv' + vu'$, $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$
When: Differentiating products and functions of functions

E A O **Inverse; parametric:** $\frac{dy}{dx} = \frac{1}{dx/dy}$; $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$
When: $\frac{dy}{dx}$ for curves given parametrically

E A O **Implicit:** $\frac{d}{dx}f(y) = f'(y)\frac{dy}{dx}$, $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$
When: Gradients of curves you cannot write as $y=\dots$

E A O **Stationary:** $f'=0$; **max if** $f''<0$, **min if** $f''>0$. **Convex** $f''>0$, **concave** $f''<0$; **inflection:** f'' **changes sign**
When: Classify stationary points; concave or convex; inflection

E A O **Tangent; normal:** $y-y_1 = f'(x_1)(x-x_1)$; $m_n = -\frac{1}{f'(x_1)}$
When: Equations of the tangent and the normal at a point

Coordinate geometry

E A O **Line; gradient:** $y-y_1 = m(x-x_1)$, $m = \frac{y_2-y_1}{x_2-x_1}$
When: Equation of a line from a gradient and a point, or two points

E A O **Perpendicular gradients:** $m_1m_2 = -1$
When: Equations of parallel and perpendicular lines

E A O **Midpoint; distance:** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
When: Distance between two points; midpoint and perpendicular bisector

E A O **Circle, centre (a,b), radius r:** $(x-a)^2+(y-b)^2 = r^2$
When: Complete the square to find the centre and radius

E A O **Circles: tangent $\perp$ radius; angle in a semicircle is 90°; perpendicular from centre bisects a chord**
When: Tangent equations; the circumcircle of a triangle

E A O **Parametric → Cartesian: eliminate t, e.g. with $\sin^2 t + \cos^2 t = 1$**
When: Convert parametric equations to Cartesian form

Numerical methods & vectors

E A O **Sign change: continuous f with $f(a)f(b)<0$ has a root in (a,b). Iteration $x_{n+1} = g(x_n)$: staircase/cobweb**
When: Locate a root by a change of sign

E A O **Magnitude; unit vector:** $|x\mathbf{i}+y\mathbf{j}+z\mathbf{k}| = \sqrt{x^2+y^2+z^2}$, $\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$
When: Magnitudes, unit vectors and angles with the axes

E A O $\overrightarrow{AB}$; **distance** AB: $|\mathbf{b}-\mathbf{a}| = \sqrt{\Delta x^2+\Delta y^2+\Delta z^2}$
When: The vector between two points and the distance between them

Statistics & probability

E A O **Mean:** $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$
When: Mean of raw and grouped data

E A O **Outliers (common rule):** $< Q_1 - 1.5(Q_3-Q_1)$ **or** $> Q_3 + 1.5(Q_3-Q_1)$
When: Identify outliers; draw box plots

E A O **Coding** $y = \frac{x-a}{b}$: $\bar{x} = a + b\bar{y}$, $\sigma_x = |b|\,\sigma_y$
When: Getting back to the original data from coded summaries

E A O **Standard normal:** $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$
When: Using the normal tables, and finding an unknown mean or standard deviation

E A O **Normal: points of inflection at $\mu \pm \sigma$; about 68%, 95%, 99.7% within 1, 2, 3 s.d.**
When: Shape and properties of the normal distribution

E A O **Normal approx. to $B(n,p)$ (n large, p near ½):** $N\left(np,\ np(1-p)\right)$
When: Approximate $B(n,p)$ when n is large and p near 0.5

E A O **Use a continuity correction, e.g.** $P(X \le 4) \approx P(Y < 4.5)$
When: Approximating a binomial with a normal

E A O **Tests: reject H_0 if p-value $<\alpha$; two-tailed: $\alpha/2$ each tail; PMCC test** $H_0: \rho = 0$
When: Binomial tests; halve the significance level for each tail

Sequences & series

E A O **Arithmetic nth term:** $u_n = a + (n-1)d$
When: Finding a term, or setting up equations from two terms

E A O **Geometric nth term:** $u_n = ar^{n-1}$
When: Finding a term, or setting up equations from two terms

E A O $(a+bx)^n = a^n(1+\frac{b}{a}x)^n$, **valid for** $|x| < \left|\frac{a}{b}\right|$
When: Take out a factor first to get the form $(1+bx)^n$

E A O **Sigma; recurrence:** $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$; $u_{n+1} = f(u_n)$
When: Sigma notation; sequences from recurrence relations

Exponentials & logarithms

E A O **Definition:** $a^x = b \iff x = \log_a b$
When: Convert between forms such as $2^5 = 32$ and $\log_2 32 = 5$

E A O **Laws:** $\log_a xy = \log_a x + \log_a y$, $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^k = k\log_a x$
When: Solving log equations and linearising data

E A O **Inverses:** $e^{\ln x} = x$, $\ln e^x = x$
When: Natural logarithms to solve equations involving e^x

E A O **Linearising:** $y = ax^n \Rightarrow \log y = \log a + n\log x$; $y = kb^x \Rightarrow \log y = \log k + x\log b$
When: Change exponential relationships into linear form

Integration

E A O **Standard:** $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$, $\int \frac{1}{x}\,dx = \ln|x| + c$, $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + c$
When: Reversing the standard derivatives

E A O **Trig:** $\int \cos kx\,dx = \frac{1}{k}\sin kx + c$, $\int \sin kx\,dx = -\frac{1}{k}\cos kx + c$
When: Reversing the standard derivatives

E A **O** **Reverse chain:** $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$, $\int f'(x)[f(x)]^n\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$
When: Any fraction whose top is (a multiple of) the derivative of the bottom

E A O **Linear inside:** $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + c$
When: Integrate e^{3x+1} or $\cos(2x-1)$ by reversing the chain rule

E A O **Substitution: $u=g(x)$, replace dx by $\frac{du}{g'(x)}$, change the limits**
When: Choose a substitution; change the limits

E A O **Area:** $\int_a^b y\,dx$; **between curves** $\int_a^b (y_{\text{top}} - y_{\text{bottom}})\,dx$; **parametric** $\int y\frac{dx}{dt}\,dt$
When: Areas under and between curves

E A O **Identities to integrate:** $\sin^2 x = \frac{1}{2}(1-\cos 2x)$, $\cos^2 x = \frac{1}{2}(1+\cos 2x)$
When: Integrating $\sin^2$ and $\cos^2$

E A O **Separable DE:** $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\,dx$
When: Separable differential equations: general and particular solutions

Trigonometry

E A O **Sine rule; cosine rule:** $\frac{a}{\sin A} = \frac{b}{\sin B}$, $a^2 = b^2 + c^2 - 2bc\cos A$
When: A side and its opposite angle (sine); two sides and the angle between (cosine)

E A O **Area; arc; sector (radians):** $\frac{1}{2}ab\sin C$; $s = r\theta$; $A = \frac{1}{2}r^2\theta$
When: Arc lengths, sector areas and segments in radians

E A O **Identities:** $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$
When: Simplify expressions and prove simple identities

E A O **Reciprocal:** $\sec^2\theta = 1 + \tan^2\theta$, $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$
When: Equations mixing $\tan^2$ and sec, or $\cot^2$ and cosec

E A O **Double angle:** $\sin 2A = 2\sin A\cos A$, $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$
When: Equations with $2x$ and x together, and integrating $\sin^2 x$

E A O **Double angle (tan):** $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$
When: Simplify expressions with $\tan 2A$

E A O **Harmonic form, $R>0$:** $a\sin\theta + b\cos\theta = R\sin(\theta+\alpha)$, $R = \sqrt{a^2+b^2}$, $\tan\alpha = \frac{b}{a}$
When: Maximum and minimum values; solving $a\sin\theta + b\cos\theta = c$

E A O **Exact values:** $\sin 30° = \frac{1}{2}$, $\sin 45° = \frac{\sqrt{2}}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$, $\tan 60° = \sqrt{3}$
When: sin, cos and tan of angles in all four quadrants

E A O **Inverse trig ranges:** $\arcsin \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\arccos \in [0, \pi]$, $\arctan \in (-\frac{\pi}{2}, \frac{\pi}{2})$
When: arcsin, arccos and arctan with restricted domains and ranges

E A O **Radians:** π **rad** $= 180°$
When: Convert between degrees and radians

Mechanics

E **A O** **... in vectors:** $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
When: Constant acceleration formulae in vector form

E A O **Variable acceleration:** $v = \frac{dr}{dt}$, $a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$, $r = \int v\,dt$, $v = \int a\,dt$
When: Motion when the acceleration is not constant (suvat does not apply)

E A O **Forces:** $F = ma$, $W = mg$, $F \le \mu R$ **(limiting:** $F = \mu R$**)**
When: Every dynamics question

E A O **Moments:** moment $= F \times d_\perp$; **equilibrium:** $\sum F = 0$, $\sum M = 0$
When: Rods on supports, ladders and tipping questions

E A O **Projectiles (speed U, angle α):** $x = Ut\cos\alpha$, $y = Ut\sin\alpha - \frac{1}{2}gt^2$
When: Time of flight, range and greatest height

E A O **Slope at θ: weight component $mg\sin\theta$ down the slope; $R = mg\cos\theta$ if nothing else acts perpendicular to it**
When: Resolve forces parallel and perpendicular to an inclined plane

E A O **Speed = $|\mathbf{v}|$; velocity–time graph: gradient = acceleration, area = displacement**
When: Displacement–time and velocity–time graphs

Independent revision resource, not endorsed by Pearson, AQA or OCR. Ticks: checked against Edexcel's and AQA's current booklets and the formulae in OCR's question papers.