S1 Discrete Random Variables: exam questions

10 questions · 30 marks · full mark schemes

Name _____ Date _____ Score / 30

- Probability distributions of discrete random variables, expectation E(X), variance Var(X), cumulative distribution functions.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/s1/discrete-random-variables-questions

1. **2 marks · warm-up**

The random variable X has a discrete uniform distribution over the integers $1, 2, 3, \ldots, 11$. Find $E(X)$.

2. **2 marks · warm-up**

Given that a random variable X has an expected value $E(X) = 5$, find $E(4 - 3X)$.

3. **3 marks · standard**

The cumulative distribution function $F(x)$ for a discrete random variable X is given by $F(1) = 0.2$, $F(2) = 0.7$, and $F(3) = 1$. Find $P(X = 2)$.

4. 3 marks · standard

The random variable X has probability function $P(X = x) = \frac{k}{x}$ for $x = 1, 2, 4$. Find the value of k.

5. 3 marks · standard

The discrete random variable X can only take the values $1, 2, 3,$ and 4. The cumulative distribution function $F(x)$ is given by the formula $F(x) = \frac{x^2}{16}$. Find $P(X = 3)$.

6. 3 marks · standard

The random variable X has probability function $P(X = x) = \frac{x^2}{k}$ for $x = 1, 2, 3$. Find the value of k.

7. 3 marks · harder

A discrete random variable X has $E(X) = 2$ and $Var(X) = 5$. Find the exact value of $E(X^2 + X)$.

8. 3 marks · harder

Two fair six-sided dice are rolled. Let the random variable Y be the absolute difference between the scores on the two dice, $Y = |X_1 - X_2|$. Find $P(Y = 0)$.

9. 4 marks · harder

A biased coin is tossed 3 times. Let X be the number of heads obtained. The probability of getting a head on a single toss is $\frac{2}{3}$. Construct the probability distribution for X and find $P(X = 2)$.

10. 4 marks · harder

X has p.m.f. $P(X = x) = kx$ for $x = 1, 2, 3, 4$. Find k and then $P(X > 2)$.

Answers and worked solutions

Question 1 — 2 marks

1. M1 for using the discrete uniform mean formula $\frac{n+1}{2}$.
2. A1 for calculating $\frac{11+1}{2} = 6$.

Question 2 — 2 marks

1. M1 for applying the linearity of expectation: $E(4 - 3X) = 4 - 3E(X)$.
2. A1 for calculating $4 - 3(5) = 4 - 15 = -11$.

Question 3 — 3 marks

1. M1 for knowing the relationship $P(X = 2) = F(2) - F(1)$.
2. M1 for substituting the values: $0.7 - 0.2$.
3. A1 for stating the probability is 0.5.

Question 4 — 3 marks

1. M1 for writing out the sum of probabilities: $k(1) + k(\frac{1}{2}) + k(\frac{1}{4}) = 1$.
2. M1 for simplifying to $k(1.75) = 1$ or $k(\frac{7}{4}) = 1$.
3. A1 for solving to get $k = \frac{4}{7}$.

Question 5 — 3 marks

1. M1 for knowing that $P(X = 3) = F(3) - F(2)$.
2. M1 for calculating $F(3) = \frac{9}{16}$ and $F(2) = \frac{4}{16}$.
3. A1 for evaluating the difference to get $\frac{5}{16}$.

Question 6 — 3 marks

1. M1 for setting up the equation $\sum P(X = x) = 1$.
2. M1 for substituting the terms: $\frac{1}{k} + \frac{4}{k} + \frac{9}{k} = 1$.
3. A1 for simplifying $\frac{14}{k} = 1$ to find $k = 14$.

Question 7 — 3 marks

1. M1 for expanding the expectation: $E(X^2 + X) = E(X^2) + E(X)$.
2. M1 for finding $E(X^2)$ using $Var(X) = E(X^2) - E(X)^2 \implies 5 = E(X^2) - 4 \implies E(X^2) = 9$.
3. A1 for summing to find the final value: $9 + 2 = 11$.

Question 8 — 3 marks

1. M1 for recognising that $Y = 0$ occurs when both dice show the same number (e.g., $(1,1), (2,2), \ldots$).
2. M1 for identifying there are 6 such outcomes out of a total of 36.
3. A1 for simplifying the probability to $\frac{6}{36} = \frac{1}{6}$.

Question 9 — 4 marks

1. M1 for recognising this uses binomial probabilities (or a tree diagram) for $X \in \{0, 1, 2, 3\}$.
2. M1 for identifying the ways to get 2 heads: HHT, HTH, THH (3 ways).
3. M1 for calculating the probability of one such arrangement: $(\frac{2}{3})^2 \times \frac{1}{3} = \frac{4}{27}$.
4. A1 for multiplying by 3 to get $P(X = 2) = 3 \times \frac{4}{27} = \frac{12}{27} = \frac{4}{9}$.

Question 10 — 4 marks

Answer: $k = 0.1$, $P(X > 2) = 0.7$

$k(1 + 2 + 3 + 4) = 1$ **M1**

$k = 0.1$ **A1**

$P(X > 2) = P(X = 3) + P(X = 4) = 3k + 4k$ **M1**

$= 0.7$ **A1**

Answer: $k = 0.1$, $P(X > 2) = 0.7$

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