P4 Vectors: exam questions

10 questions · 39 marks · full mark schemes

Name _____ Date _____ Score / 39

- Vectors in 2D and 3D; magnitude, unit vectors, scalar product and angles; vector equation of a line; intersecting, parallel and skew lines.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p4/vectors-questions

1. **2 marks · warm-up**

Find the exact magnitude of the vector $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$.

2. **2 marks · warm-up**

The position vectors of points A and B are $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$. Find the vector $\overrightarrow{AB}$.

3. **2 marks · standard**

Find a vector equation of the line passing through the points $A(1, -2, 4)$ and $B(3, 1, 5)$.

4. 3 marks · standard

Let $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Find the exact value of $|2\mathbf{a} - \mathbf{b}|$.

5. 3 marks · standard

Determine whether the point $(7, 5, -2)$ lies on the line with vector equation $\mathbf{r} = (\mathbf{i} - \mathbf{j} + 4\mathbf{k}) + t(2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

6. 3 marks · standard

The points A, B, and C have position vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ respectively relative to origin O. Point M is the midpoint of BC. Prove that $\overrightarrow{AM} = \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c} - \mathbf{a}$.

7. 4 marks · harder

Two particles, P_1 and P_2, are moving such that their position vectors at time t are $\mathbf{r}_1 = (2\mathbf{i} + \mathbf{k}) + t(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r}_2 = (-4\mathbf{j} - 3\mathbf{k}) + t(2\mathbf{i} + 4\mathbf{j} + \mathbf{k})$ respectively. Show that the particles collide and find the position vector of the collision.

8. 5 marks · harder

The vertices of a triangle are $A(1,0,0)$, $B(0,1,0)$, and $C(0,0,1)$. Find the exact area of the triangle.

9. 5 marks · harder

The line ℓ has vector equation $\mathbf{r} = (1,2,3) + t(2,-1,1)$. Find the shortest distance from the point $P(4,3,1)$ to ℓ.

10. 10 marks · harder

Consider a rhombus $ABCD$. Let the vectors representing two adjacent sides be $\vec{AB} = \mathbf{a}$ and $\vec{AD} = \mathbf{b}$.

Since the shape is a rhombus, the magnitudes of these sides are equal ($|\mathbf{a}| = |\mathbf{b}|$).

Express the diagonals $\vec{AC}$ and $\vec{BD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Hence, use the scalar (dot) product to prove algebraically that the diagonals of any rhombus intersect at right angles.

Answers and worked solutions

Question 1 — 2 marks

1. M1 for applying the 3D magnitude formula: $\sqrt{3^2 + (-4)^2 + 12^2}$.
2. A1 for evaluating to exactly 13.

Question 2 — 2 marks

1. M1 for using $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.
2. A1 for the correct vector: $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$.

Question 3 — 2 marks

1. M1 for finding the direction vector $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.
2. A1 for formulating the full line equation, e.g., $\mathbf{r} = (\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}) + t(2\mathbf{i} + 3\mathbf{j} + \mathbf{k})$.

Question 4 — 3 marks

1. M1 for evaluating the linear combination $2\mathbf{a} - \mathbf{b} = 2(\mathbf{i} + \mathbf{j}) - (2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$.
2. M1 for finding the resulting vector: $0\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$.
3. A1 for calculating its magnitude: $\sqrt{0^2 + 3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$.

Question 5 — 3 marks

1. M1 for equating components: $1 + 2t = 7$, $-1 + 2t = 5$, and $4 - 2t = -2$.
2. M1 for solving the first equation to find $t = 3$.
3. A1 for verifying $t = 3$ holds for the y and z components ($-1 + 6 = 5$ and $4 - 6 = -2$), concluding that the point does lie on the line.

Question 6 — 3 marks

1. M1 for identifying the position vector of the midpoint $\overrightarrow{OM} = \frac{1}{2}(\mathbf{b} + \mathbf{c})$.
2. M1 for using vector subtraction $\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA}$.
3. A1 for substituting terms to complete the proof: $\frac{1}{2}(\mathbf{b} + \mathbf{c}) - \mathbf{a} = \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c} - \mathbf{a}$.

Question 7 — 4 marks

1. M1 for equating the position vectors to find the time of collision: $2 + t = 2t$, $2t = -4 + 4t$, and $1 - t = -3 + t$.
2. M1 for solving one of the equations to find t, e.g., $2 + t = 2t \implies t = 2$.
3. M1 for verifying $t = 2$ satisfies the other components (e.g., $2(2) = -4 + 8$ and $1 - 2 = -3 + 2$), confirming a collision.
4. A1 for substituting $t = 2$ to find the collision position vector: $4\mathbf{i} + 4\mathbf{j} - \mathbf{k}$.

Question 8 — 5 marks

1. M1 for finding two vectors forming the triangle, e.g., $\overrightarrow{AB} = -\mathbf{i} + \mathbf{j}$ and $\overrightarrow{AC} = -\mathbf{i} + \mathbf{k}$.
2. M1 for calculating their magnitudes: $|\overrightarrow{AB}| = \sqrt{2}$ and $|\overrightarrow{AC}| = \sqrt{2}$.
3. M1 for calculating the dot product $\overrightarrow{AB} \cdot \overrightarrow{AC} = (-1)(-1) + (1)(0) + (0)(1) = 1$ to find the angle.
4. M1 for finding $\cos\theta = \frac{1}{2}$, so $\theta = 60°$, and using Area $= \frac{1}{2}|AB||AC|\sin\theta$.
5. A1 for the final exact area: $\frac{1}{2}(\sqrt{2})(\sqrt{2})\sin 60° = \frac{\sqrt{3}}{2}$.

Question 9 — 5 marks

Answer: $\frac{5\sqrt{2}}{2} \approx 3.54$

general point on ℓ: $X(1 + 2t,\ 2 - t,\ 3 + t)$, so

$\overrightarrow{PX} = (2t - 3,\ -1 - t,\ 2 + t)$ **M1**

$\overrightarrow{PX} \cdot (2, -1, 1) = 0 \Rightarrow 6t - 3 = 0$ **M1**

$t = \frac{1}{2}$ **A1**

$\overrightarrow{PX} = (-2,\ -1.5,\ 2.5)$,

$|\overrightarrow{PX}| = \sqrt{4 + 2.25 + 6.25}$ **M1**

$= \sqrt{12.5} = \frac{5\sqrt{2}}{2} \approx 3.54$ **A1**

Answer: $\frac{5\sqrt{2}}{2} \approx 3.54$

Question 10

10 marks

Answer: $\overrightarrow{AC} = \mathbf{a} + \mathbf{b}$, $\overrightarrow{BD} = \mathbf{b} - \mathbf{a}$, and $\overrightarrow{AC} \cdot \overrightarrow{BD} = |\mathbf{b}|^2 - |\mathbf{a}|^2 = 0$, so the diagonals are perpendicular

By vector addition rules, the diagonal $\vec{AC} = \vec{AB} + \vec{BC}$. Since $\vec{BC}$ is parallel to $\vec{AD}$, $\vec{AC} = \mathbf{a} + \mathbf{b}$. $M1$**A1**

The other diagonal $\vec{BD} = \vec{BA} + \vec{AD} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$. $M1$**A1**

To prove they are perpendicular, we must show their dot product equals 0. **M1**

$\vec{AC} \cdot \vec{BD} = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{b} - \mathbf{a})$. **M1**

Expand the brackets: $\mathbf{a} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{a} + \mathbf{b} \cdot \mathbf{b} - \mathbf{b} \cdot \mathbf{a}$. $M1$**A1**

Since dot products are commutative ($\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$), the middle terms cancel out. We are left with $|\mathbf{b}|^2 - |\mathbf{a}|^2$. **M1**

Since a rhombus has equal side lengths, $|\mathbf{a}| = |\mathbf{b}|$, so $|\mathbf{b}|^2 - |\mathbf{a}|^2 = 0$. **B1**

Because the dot product is exactly zero, the diagonals intersect at right angles.

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