# P4 Partial Fractions: exam questions

10 questions · 40 marks · full mark schemes

Name _____ Date _____ Score / 40

- Partial fractions with distinct linear and repeated linear factors in the denominator (improper fractions by division first). Irreducible-quadratic denominators are not on the IAL specification.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p4/partial-fractions-questions

**1.** **3 marks · warm-up**

Express $\dfrac{3}{(x)(x+3)}$ in partial fractions.

**2.** **5 marks · warm-up**

Find the exact values of the constants $A$ and $B$ in the partial fraction decomposition $\frac{5x-3}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$.

**3.** **3 marks · standard**

Express $\frac{7x-1}{(2x+1)(x-1)}$ in partial fractions.

**4.** 3 marks · standard

Express $\frac{5}{(x-2)(x+3)}$ in partial fractions.

**5.** 4 marks · standard

Express $\frac{x^2+3x-2}{x^2-1}$ in partial fractions.

**6.** 5 marks · standard

Express $\frac{2x^2-3x+1}{x(x-1)^2}$ in partial fractions.

**7.** 4 marks · harder

Express $\frac{2x^2-5x-9}{(x+2)(x^2-1)}$ in partial fractions.

**8.** **4 marks · harder**

Express $\frac{-x-7}{(x+1)^2(x-2)}$ in partial fractions.

**9.** **4 marks · harder**

Express $\frac{4x^2}{(x-1)^2(x+1)}$ in partial fractions.

**10.** **5 marks · harder**

Express $\frac{x^3+3x^2-3x-6}{x^2-4}$ in the form $Ax + B + \frac{C}{x-2} + \frac{D}{x+2}$.

# Answers and worked solutions

## Question 1

**3 marks**

**Answer:** $\frac{1}{x} - \frac{1}{x+3}$

$\frac{3}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$ **M1**. $3 = A(x+3) + Bx$; $x = 0$: $A = 1$; $x = -3$: $B = -1$ **A1 A1**.

## Question 2

**5 marks**

**Answer:** $A = 2, B = 3$

M1 for forming the identity:
$5x - 3 = A(x-3) + B(x+1)$.
M1 for substituting a strategic value, e.g. $x = 3$:
$5(3) - 3 = A(0) + B(4) \implies 12 = 4B$.
A1 for exactly $B = 3$.
M1 for substituting a second strategic value, e.g. $x = -1$:
$5(-1) - 3 = A(-4) + B(0) \implies -8 = -4A$.
A1 for exactly $A = 2$.

## Question 3

**3 marks**

1. M1 for setting up $\frac{A}{2x+1} + \frac{B}{x-1}$ and forming $A(x-1) + B(2x+1) = 7x - 1$.
2. M1 for substituting $x = 1$ to find $3B = 6 \implies B = 2$.
3. A1 for substituting $x = -0.5$ to find $-1.5A = -4.5 \implies A = 3$, giving $\frac{3}{2x+1} + \frac{2}{x-1}$.

## Question 4

**3 marks**

1. M1 for forming the identity $A(x+3) + B(x-2) = 5$.
2. M1 for solving for $A$ by setting $x = 2 \implies 5A = 5 \implies A = 1$.
3. A1 for solving for $B$ by setting $x = -3 \implies -5B = 5 \implies B = -1$, giving $\frac{1}{x-2} - \frac{1}{x+3}$.

## Question 5

**4 marks**

1. M1 for performing algebraic long division (or equivalent method) to extract the integer part: $1$ with a remainder of $3x - 1$.
2. M1 for forming the identity $\frac{3x-1}{(x-1)(x+1)} \equiv \frac{A}{x-1} + \frac{B}{x+1}$.
3. M1 for finding the constants $A = 1$ and $B = 2$.
4. A1 for combining everything into the final expression: $1 + \frac{1}{x-1} + \frac{2}{x+1}$.

## Question 6

**5 marks**

**Answer:** $\frac{1}{x} + \frac{1}{x-1}$

$\frac{2x^2-3x+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ **M1**.
$2x^2 - 3x + 1 = A(x-1)^2 + Bx(x-1) + Cx$ **M1**.
$x = 0$: $A = 1$ **A1**. $x = 1$: $0 = C \Rightarrow C = 0$ **A1**. Coeff of $x^2$: $2 = A + B \Rightarrow B = 1$ **A1**.

## Question 7

**4 marks**

1. M1 for completely factorising the denominator to $(x+2)(x-1)(x+1)$ and forming $A(x-1)(x+1) + B(x+2)(x+1) + C(x+2)(x-1) = 2x^2 - 5x - 9$.
2. M1 for substituting $x = -2$ to get $3A = 9 \implies A = 3$.
3. M1 for substituting $x = 1$ to get $6B = -12 \implies B = -2$.
4. A1 for substituting $x = -1$ to get $-2C = -2 \implies C = 1$, giving $\frac{3}{x+2} - \frac{2}{x-1} + \frac{1}{x+1}$.

## Question 8

**4 marks**

1. M1 for setting up the correct identity format: $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-2}$, leading to $A(x+1)(x-2) + B(x-2) + C(x+1)^2 = -x - 7$.
2. M1 for substituting $x = -1$ to find $-3B = -6 \implies B = 2$.
3. M1 for substituting $x = 2$ to find $9C = -9 \implies C = -1$.
4. A1 for comparing $x^2$ coefficients to find $A + C = 0 \implies A = 1$, giving $\frac{1}{x+1} + \frac{2}{(x+1)^2} - \frac{1}{x-2}$.

## Question 9

**4 marks**

1. M1 for setting up $A(x-1)(x+1) + B(x+1) + C(x-1)^2 = 4x^2$.
2. M1 for substituting $x = 1$ to find $2B = 4 \implies B = 2$.
3. M1 for substituting $x = -1$ to find $4C = 4 \implies C = 1$.
4. A1 for comparing $x^2$ coefficients to find $A + C = 4 \implies A = 3$, giving $\frac{3}{x-1} + \frac{2}{(x-1)^2} + \frac{1}{x+1}$.

## Question 10

**5 marks**

1. M1 for performing long division to divide by $x^2 - 4$, obtaining a quotient of $x + 3$.
2. M1 for finding the correct remainder of $x + 6$.
3. M1 for writing $\frac{x+6}{(x-2)(x+2)} \equiv \frac{C}{x-2} + \frac{D}{x+2}$.
4. M1 for finding the constants $C = 2$ and $D = -1$.
5. A1 for the full expression: $x + 3 + \frac{2}{x-2} - \frac{1}{x+2}$.

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