P4 Integration: exam questions

9 questions · 44 marks · full mark schemes

Name _____ Date _____ Score / 44

- Integration by substitution, by parts and using partial fractions; trig identities in integration; volumes of revolution about the x-axis.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p4/integration-questions

1. **4 marks · warm-up**

Find $\int xe^{3x}\,dx$.

2. **5 marks · warm-up**

Find $\int \frac{2}{(x-1)(x+1)}\,dx$.

3. 4 marks · standard

The region bounded by the curve $y = \sec x$, the x-axis, and the lines $x = 0$ and $x = \frac{\pi}{4}$ is rotated $360°$ about the x-axis. Find the exact volume of the solid generated.

4. 4 marks · standard

Use the trapezium rule with 4 strips to estimate the value of $\int_0^1 \frac{1}{1+x^2}\,dx$. Give your answer to 3 decimal places.

5. 5 marks · standard

Find the exact area enclosed between the curve $y = x\ln x$, the x-axis, and the lines $x = 1$ and $x = e$.

6. 5 marks · harder

Find the particular solution to the differential equation $\frac{dy}{dx} = e^{x-y}$, given that $y = \ln 2$ when $x = 0$.

7. 5 marks · harder

Use the substitution $x = 2\sin\theta$ to find the exact value of $\int_0^1 \sqrt{4 - x^2}\,dx$.

8. 6 marks · harder

Find the exact value of $\int_0^{\pi} x^2 \sin x\,dx$.

9. 6 marks · harder

A curve is defined by the parametric equations $x = \theta - \sin\theta$ and $y = 1 - \cos\theta$. Find the exact area under one arch of the curve, bounded by the x-axis between $\theta = 0$ and $\theta = 2\pi$.

Answers and worked solutions

Question 1

4 marks

1. M1 for choosing parts: $u = x$ and $\frac{dv}{dx} = e^{3x}$.
2. M1 for finding $\frac{du}{dx} = 1$ and $v = \frac{1}{3}e^{3x}$.
3. M1 for applying the integration by parts formula: $\frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}\,dx$.
4. A1 for the final integrated answer $\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$.

Question 2

5 marks

1. M1 for expressing the integrand as partial fractions: $\frac{A}{x-1} + \frac{B}{x+1}$.
2. M1 for finding the constants $A = 1$ and $B = -1$.
3. M1 for integrating $\frac{1}{x-1}$ to $\ln|x-1|$.
4. M1 for integrating $-\frac{1}{x+1}$ to $-\ln|x+1|$.
5. A1 for combining into the final answer $\ln\left|\frac{x-1}{x+1}\right| + c$.

Question 3

4 marks

1. M1 for writing the volume integral as $\pi\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$.
2. M1 for recognising that the integral of $\sec^2 x$ is $\tan x$.
3. M1 for substituting the upper limit to get $\tan\left(\frac{\pi}{4}\right) = 1$.
4. A1 for evaluating to the final exact volume $\pi(1 - 0) = \pi$.

Question 4

4 marks

1. M1 for identifying the step size $h = 0.25$.
2. M1 for correctly calculating the y—values at $x = 0, 0.25, 0.5, 0.75, 1$: 1, $\frac{16}{17}$, 0.8, $\frac{16}{25}$, 0.5.
3. M1 for substituting into the trapezium rule formula: $\frac{0.25}{2}[1 + 0.5 + 2(\frac{16}{17} + 0.8 + 0.64)]$.
4. A1 for evaluating the expression correctly to 0.783.

Question 5

5 marks

1. M1 for recognising the need to evaluate the definite integral $\int_1^e x\ln x\,dx$.
2. M1 for applying integration by parts with $u = \ln x$ and $dv = x\,dx$.
3. M1 for obtaining the expression $\frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,dx$ and completing integration to $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2$.
4. M1 for correctly substituting the limits e and 1.
5. A1 for calculating the exact final area $\frac{1}{4}e^2 + \frac{1}{4}$.

Question 6

5 marks

1. M1 for rewriting the equation using index laws as $\frac{dy}{dx} = \frac{e^x}{e^y}$.
2. M1 for separating variables: $\int e^y\,dy = \int e^x\,dx$.
3. M1 for integrating to get $e^y = e^x + c$.
4. M1 for using the boundary conditions to find c: $e^{\ln 2} = e^0 + c \implies 2 = 1 + c \implies c = 1$.
5. A1 for writing the final solution as $y = \ln(e^x + 1)$.

Question 7

5 marks

1. M1 for differentiating to get $dx = 2\cos\theta\,d\theta$ and identifying the new limits $\theta = 0$ to $\theta = \frac{\pi}{6}$.
2. M1 for substituting into the integral to get $\int_0^{\frac{\pi}{6}} \sqrt{4 - 4\sin^2\theta}(2\cos\theta)\,d\theta$.
3. M1 for simplifying the integrand using $1 - \sin^2\theta = \cos^2\theta$ to get $\int_0^{\frac{\pi}{6}} 4\cos^2\theta\,d\theta$.
4. M1 for using the double angle identity $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ and integrating to get $2[\theta + \frac{1}{2}\sin 2\theta]_0^{\frac{\pi}{6}}$.
5. A1 for evaluating the limits to obtain the exact value $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$.

Question 8

6 marks

1. M1 for setting up the first integration by parts with $u = x^2$ and $dv = \sin x\,dx$.
2. M1 for obtaining $-x^2\cos x + \int 2x\cos x\,dx$.
3. M1 for applying integration by parts a second time on $\int 2x\cos x\,dx$ with $u = 2x$ and $dv = \cos x\,dx$.
4. A1 for the fully integrated expression $-x^2\cos x + 2x\sin x + 2\cos x$.
5. M1 for substituting the limits 0 and π: $(-\pi^2(-1) + 0 + 2(-1)) - (0 + 0 + 2)$.
6. A1 for calculating the exact final answer $\pi^2 - 4$.

Question 9

6 marks

1. M1 for setting up the area integral $\int y\frac{dx}{d\theta}\,d\theta = \int_0^{2\pi}(1 - \cos\theta)(1 - \cos\theta)\,d\theta$.
2. M1 for expanding the integrand to $1 - 2\cos\theta + \cos^2\theta$.
3. M1 for using the double angle identity to replace $\cos^2\theta$ with $\frac{1}{2} + \frac{1}{2}\cos 2\theta$.
4. M1 for forming the integrable expression $\frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos 2\theta$.
5. M1 for integrating to $\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta$.
6. A1 for evaluating the limits 0 and 2π to obtain the final exact area 3π.

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