P4 Differentiation: exam questions

10 questions · 34 marks · full mark schemes

Name _____ Date _____ Score / 34

- Implicit and parametric differentiation; connected rates of change.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p4/differentiation-questions

1. **2 marks · warm-up**

A particle moves in a straight line. Its acceleration a, which is the rate of change of its velocity v with respect to time t, is directly proportional to its velocity. Write down a differential equation for this situation.

2. **3 marks · warm-up**

Find an expression for $\frac{dy}{dx}$ in terms of x and y given that $x^2 + y^2 - 4x + 6y = 0$.

3. **3 marks · standard**

Find $\frac{dy}{dx}$ in terms of t for the curve defined by $x = e^t$ and $y = e^{2t} + t$.

4. 3 marks · standard

A curve is defined by the equation $\sin(x) + \cos(y) = 0.5$. Find an expression for $\frac{dy}{dx}$.

5. 3 marks · standard

The side length x of a cube is increasing at a rate of $0.1\ \text{cm s}^{-1}$. Find the rate at which the volume V is increasing when $x = 4\ \text{cm}$.

6. 4 marks · standard

A spherical balloon is deflating such that its volume V decreases at a constant rate of $5\ \text{cm}^3\text{s}^{-1}$. Find the rate of change of its radius r at the instant when its volume is $36\pi\ \text{cm}^3$.

7. 4 marks · harder

The curve C is defined by the parametric equations $x = t^3 - 3t$ and $y = t^2$. Find the exact coordinates of the points where the tangent to the curve is vertical.

8. 4 marks · harder

Find $\frac{dy}{dx}$ in its simplest form for the curve defined by $x = \frac{t}{1+t}$ and $y = t^2$.

9. 4 marks · harder

Water is poured into a cylindrical container of radius $10\ \text{cm}$ at a constant rate of $50\ \text{cm}^3\text{s}^{-1}$. Let h be the depth of the water. Find the rate at which the depth of the water is increasing.

10. 4 marks · harder

The area A of a circle increases at a rate that is inversely proportional to its radius r. By using the chain rule, show that the radius r increases at a rate that is inversely proportional to r^2.

Answers and worked solutions

Question 1

2 marks

1. M1 for recognising that acceleration is $\frac{dv}{dt}$.
2. A1 for formulating the differential equation $\frac{dv}{dt} = kv$, where k is a constant of proportionality.

Question 2

3 marks

1. M1 for differentiating implicitly to get $2x + 2y\frac{dy}{dx} - 4 + 6\frac{dy}{dx} = 0$.
2. M1 for collecting terms with $\frac{dy}{dx}$: $\frac{dy}{dx}(2y + 6) = 4 - 2x$.
3. A1 for the final simplified expression $\frac{dy}{dx} = \frac{2-x}{y+3}$.

Question 3

3 marks

1. M1 for differentiating to obtain $\frac{dx}{dt} = e^t$ and $\frac{dy}{dt} = 2e^{2t} + 1$.
2. M1 for dividing $\frac{dy}{dt}$ by $\frac{dx}{dt}$ to find $\frac{dy}{dx} = \frac{2e^{2t}+1}{e^t}$.
3. A1 for a correctly simplified expression, e.g., $2e^t + e^{-t}$.

Question 4

3 marks

1. M1 for differentiating $\sin(x)$ to $\cos(x)$.
2. M1 for differentiating $\cos(y)$ implicitly to $-\sin(y)\frac{dy}{dx}$ and setting the derivative of 0.5 to 0.
3. A1 for isolating $\frac{dy}{dx}$ to get $\frac{dy}{dx} = \frac{\cos(x)}{\sin(y)}$.

Question 5

3 marks

1. M1 for using $V = x^3$ to find $\frac{dV}{dx} = 3x^2$.
2. M1 for establishing the connected rate of change $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3x^2(0.1)$.
3. A1 for substituting $x = 4$ to obtain $3(16)(0.1) = 4.8\ \text{cm}^3\text{s}^{-1}$.

Question 6

4 marks

1. M1 for setting $V = \frac{4}{3}\pi r^3 = 36\pi$ and solving to find the current radius $r = 3$.
2. M1 for finding $\frac{dV}{dr} = 4\pi r^2$.
3. M1 for applying the chain rule: $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} = -5 \times \frac{1}{4\pi r^2}$.
4. A1 for substituting $r = 3$ to get $-\frac{5}{36\pi}\ \text{cm s}^{-1}$.

Question 7

4 marks

1. M1 for identifying that the tangent is vertical when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$.
2. M1 for calculating $\frac{dx}{dt} = 3t^2 - 3$ and setting it to zero to find $t = 1$ and $t = -1$.
3. M1 for substituting these values of t back into the expressions for x and y.
4. A1 for the correct coordinates: $(-2, 1)$ and $(2, 1)$.

Question 8

4 marks

1. M1 for using the quotient rule or product rule on x to find $\frac{dx}{dt} = \frac{1(1+t)-t(1)}{(1+t)^2} = \frac{1}{(1+t)^2}$.
2. M1 for finding $\frac{dy}{dt} = 2t$.
3. M1 for dividing $\frac{dy}{dt}$ by $\frac{dx}{dt}$ to get $\frac{dy}{dx} = 2t \div \frac{1}{(1+t)^2}$.
4. A1 for the simplified final expression $2t(1+t)^2$.

Question 9

4 marks

1. M1 for defining the volume $V = \pi r^2 h$.
2. M1 for substituting the constant radius $r = 10$ to get $V = 100\pi h$, so $\frac{dV}{dh} = 100\pi$.
3. M1 for applying the chain rule $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = 50 \times \frac{1}{100\pi}$.
4. A1 for the exact final rate $\frac{1}{2\pi}\ \text{cm s}^{-1}$.

Question 10

4 marks

1. M1 for setting up the differential equation $\frac{dA}{dt} = \frac{k}{r}$, where k is a constant.
2. M1 for finding $\frac{dA}{dr} = 2\pi r$ from $A = \pi r^2$.
3. M1 for applying the chain rule $\frac{dr}{dt} = \frac{dA}{dt} \div \frac{dA}{dr} = \frac{k}{r} \div 2\pi r$.
4. A1 for concluding $\frac{dr}{dt} = \frac{k}{2\pi r^2} = \frac{C}{r^2}$ (where $C = \frac{k}{2\pi}$ is another constant), demonstrating the inverse proportionality to r^2.

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