P4 Coordinate Geometry: exam questions

10 questions · 35 marks · full mark schemes

Name _____ Date _____ Score / 35

- Parametric curves and conversion to Cartesian form.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p4/coordinate-geometry-questions

1. **3 marks · warm-up**

A curve has parametric equations $x = 4\cos\theta$ and $y = 4\sin\theta$. Find the Cartesian equation of the curve.

2. **3 marks · standard**

A curve is defined by $x = t^2 - 4$ and $y = t^3 - t$. Find the coordinates of the points where the curve crosses the y-axis.

3. **3 marks · standard**

A particle moves such that its position is given by $x = 4t$ and $y = \frac{12}{t}$. Find the shortest distance from the particle to the origin for $t > 0$.

4. 3 marks · standard

Find the Cartesian equation of the curve $x = \sin t + 2$ and $y = \operatorname{cosec} t$.

5. 4 marks · standard

Find the Cartesian equation for the curve defined parametrically by $x = \frac{2}{t}$ and $y = 3t^2 - t$.

6. 3 marks · harder

A curve is defined parametrically by $x = \cot t$ and $y = \operatorname{cosec}^2 t$. Find the Cartesian equation of the curve.

7. 4 marks · harder

A curve has parametric equations $x = 2t$ and $y = 3t^2 - 1$. Find the coordinates of the points where the curve intersects the line $y = 5x - 1$.

8. 4 marks · harder

The parametric equations of a curve are $x = \ln(t+1)$ and $y = \ln(t^2-1)$ for $t > 1$. Find the Cartesian equation of the curve in the form $y = f(x)$.

9. 4 marks · harder

Find the Cartesian equation of the curve given by $x = \frac{t-1}{t+1}$ and $y = \frac{t+1}{t-1}$.

10. 4 marks · harder

The curve C is defined by $x = e^{3t}$ and $y = e^{6t} - e^{3t}$. Show that C is part of a parabola and find its Cartesian equation.

Answers and worked solutions

Question 1

3 marks

1. M1 for rearranging to $\cos\theta = \frac{x}{4}$ and $\sin\theta = \frac{y}{4}$.
2. M1 for substituting into the identity $\cos^2\theta + \sin^2\theta = 1$.
3. A1 for obtaining $(\frac{x}{4})^2 + (\frac{y}{4})^2 = 1 \implies x^2 + y^2 = 16$.

Question 2

3 marks

1. M1 for recognising that the curve crosses the y-axis when $x = 0$, so $t^2 - 4 = 0$.
2. M1 for finding the parameter values $t = 2$ and $t = -2$.
3. A1 for substituting these back into y to get $y = 8 - 2 = 6$ and $y = -8 - (-2) = -6$, yielding coordinates $(0, 6)$ and $(0, -6)$.

Question 3

3 marks

1. M1 for forming the Cartesian equation $y = \frac{12}{x/4} = \frac{48}{x}$, or for setting up the distance squared $D^2 = x^2 + y^2 = 16t^2 + \frac{144}{t^2}$.
2. M1 for using differentiation $\frac{d(D^2)}{dt} = 32t - \frac{288}{t^3} = 0 \implies t^4 = 9 \implies t = \sqrt{3}$ (since $t > 0$).
3. A1 for evaluating the minimum distance: $D^2 = 16(3) + \frac{144}{3} = 48 + 48 = 96 \implies D = \sqrt{96} = 4\sqrt{6}$.

Question 4

3 marks

1. M1 for isolating $\sin t$: $\sin t = x - 2$.
2. M1 for writing $y = \frac{1}{\sin t}$.
3. A1 for the final Cartesian equation $y = \frac{1}{x-2}$.

Question 5

4 marks

1. M1 for rearranging to find t in terms of x: $t = \frac{2}{x}$.
2. M1 for substituting t into y: $y = 3(\frac{2}{x})^2 - (\frac{2}{x})$.
3. M1 for expanding the terms: $y = 3(\frac{4}{x^2}) - \frac{2}{x}$.
4. A1 for the simplified Cartesian equation $y = \frac{12}{x^2} - \frac{2}{x}$.

Question 6

3 marks

1. M1 for recalling the trigonometric identity $\operatorname{cosec}^2 t = 1 + \cot^2 t$.
2. M1 for substituting $x = \cot t$ and $y = \operatorname{cosec}^2 t$ into the identity.
3. A1 for the Cartesian equation $y = 1 + x^2$.

Question 7

4 marks

1. M1 for substituting the parametric equations into the line equation: $3t^2 - 1 = 5(2t) - 1$.
2. M1 for rearranging to form a quadratic in t: $3t^2 - 10t = 0$.
3. M1 for solving to find $t = 0$ and $t = \frac{10}{3}$.
4. A1 for substituting back to find the intersection points: $(0, -1)$ and $(\frac{20}{3}, \frac{97}{3})$.

Question 8

4 marks

1. M1 for expressing t in terms of x: $e^x = t + 1 \implies t = e^x - 1$.
2. M1 for using logarithmic laws to simplify y: $y = \ln((t-1)(t+1))$.
3. M1 for substituting $t = e^x - 1$: $y = \ln((e^x - 2)(e^x))$.
4. A1 for expanding or leaving in a simplified exact form: $y = \ln(e^x - 2) + x$.

Question 9

4 marks

1. M1 for observing the relationship between x and y: y is the reciprocal of x.
2. M1 for establishing $y = \frac{1}{x}$.
3. A1 for stating the final Cartesian equation $y = x^{-1}$ or $xy = 1$.
4. A1 for specifying the domain $x \neq 1$ (since $t \neq -1$ and $t \neq 1$).

Question 10

4 marks

1. M1 for rewriting y in terms of e^{3t}: $y = (e^{3t})^2 - e^{3t}$.
2. M1 for substituting $x = e^{3t}$ directly into the equation for y.
3. A1 for obtaining the Cartesian equation $y = x^2 - x$.
4. A1 for stating that the domain is $x > 0$ (since $e^{3t} > 0$), making it part of a parabola.

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