P4 The Binomial Expansion: exam questions

10 questions · 38 marks · full mark schemes

Name _____ Date _____ Score / 38

- Expanding $(a + bx)^n$ for any rational n; validity $|bx/a| < 1$; using partial fractions with expansions.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p4/binomial-expansion-questions

1. **2 marks · warm-up**

State the range of values of x for which the binomial expansion of $(1 - 5x)^{-3}$ is valid.

2. **2 marks · standard**

State the range of values of x for which the expansion of $(1 - x)(2 + 3x)^{-1}$ is valid.

3. **3 marks · standard**

Given that the coefficient of x^2 in the expansion of $(1 + kx)^{-3}$ is 54, find the possible values of the constant k.

4. 4 marks · standard

By expanding $(1-x)^{\frac{1}{2}}$ up to the term in x^2, find an approximation for $\sqrt{0.96}$. Give your answer to 4 decimal places.

5. 4 marks · standard

Find the first three terms in the binomial expansion of $(4-x)^{\frac{1}{2}}$.

6. 4 marks · standard

Find the binomial expansion of $(1+2x)(1-x)^{-1}$ in ascending powers of x, up to and including the term in x^2.

7. 4 marks · harder

Find the exact coefficient of x^3 in the binomial expansion of $(8+x)^{\frac{1}{3}}$.

8. 5 marks · harder

Find the expansion of $(3-x)(1+2x)^{-2}$ in ascending powers of x, up to and including the term in x^2.

9. 5 marks · harder

Find the expansion of $\frac{2+x}{\sqrt{1-x}}$ in ascending powers of x, up to and including the term in x^2.

10. 5 marks · harder

Using partial fractions, or otherwise, find the series expansion of $\frac{3}{(1-x)(1+2x)}$ up to and including the term in x^2.

Answers and worked solutions

Question 1 — 2 marks

1. M1 for setting up the inequality $|-5x| < 1$.
2. A1 for correctly isolating x to state $|x| < \frac{1}{5}$ (or $-\frac{1}{5} < x < \frac{1}{5}$).

Question 2 — 2 marks

1. M1 for identifying that the validity solely depends on the term $(2+3x)^{-1} = 2^{-1}(1+\frac{3x}{2})^{-1}$.
2. A1 for concluding that $|\frac{3x}{2}| < 1 \implies |x| < \frac{2}{3}$.

Question 3 — 3 marks

1. M1 for setting up the x^2 term: $\frac{(-3)(-4)}{2!}(kx)^2$.
2. M1 for equating the coefficient to 54:
   $6k^2 = 54 \implies k^2 = 9$.
3. A1 for finding both possible values: $k = 3$ and $k = -3$.

Question 4 — 4 marks

1. M1 for finding the expansion:
   $(1-x)^{\frac{1}{2}} \approx 1 - \frac{1}{2}x - \frac{1}{8}x^2$.
2. M1 for identifying the correct substitution $x = 0.04$.
3. M1 for substituting 0.04 into the expanded expression: $1 - \frac{1}{2}(0.04) - \frac{1}{8}(0.0016)$.
4. A1 for calculating $1 - 0.02 - 0.0002 = 0.9798$.

Question 5 — 4 marks

1. M1 for factoring out $4^{\frac{1}{2}}$: $2(1-\frac{x}{4})^{\frac{1}{2}}$.
2. M1 for expanding
   $(1-\frac{x}{4})^{\frac{1}{2}} \approx 1 + \frac{1}{2}(-\frac{x}{4}) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-\frac{x}{4})^2$.
3. M1 for simplifying the brackets to $1 - \frac{x}{8} - \frac{x^2}{128}$.
4. A1 for multiplying by 2 to get $2 - \frac{x}{4} - \frac{x^2}{64}$.

Question 6 — 4 marks

1. M1 for expanding $(1-x)^{-1} = 1 + x + x^2 + \ldots$.
2. M1 for setting up the product:
   $(1+2x)(1+x+x^2)$.
3. M1 for systematically expanding the brackets:
   $1(1+x+x^2) + 2x(1+x)$.
4. A1 for collecting like terms to get $1 + 3x + 3x^2$.

Question 7 — 4 marks

1. M1 for rewriting as $8^{\frac{1}{3}}(1+\frac{x}{8})^{\frac{1}{3}} = 2(1+\frac{x}{8})^{\frac{1}{3}}$.
2. M1 for expressing the x^3 term formula:
   $2 \times \left[\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(\frac{x}{8})^3\right]$.
3. M1 for evaluating the combination of fractions:
   $2 \times \frac{10/27}{6} \times \frac{x^3}{512}$.
4. A1 for the simplified exact coefficient: $\frac{5}{20736}$.

Question 8 — 5 marks

1. M1 for expanding
   $(1+2x)^{-2} \approx 1 + (-2)(2x) + \frac{(-2)(-3)}{2!}(2x)^2$.
2. A1 for simplifying the inner expansion correctly to
   $1 - 4x + 12x^2$.
3. M1 for setting up the multiplication:
   $(3-x)(1-4x+12x^2)$.
4. M1 for distributing the terms up to x^2:
   $3(1-4x+12x^2) - x(1-4x)$.
5. A1 for the fully simplified expression:
   $3 - 13x + 40x^2$.

Question 9 — 5 marks

1. M1 for rewriting the expression as
   $(2+x)(1-x)^{-\frac{1}{2}}$.
2. M1 for expanding $(1-x)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}x + \frac{3}{8}x^2$.
3. M1 for setting up the product:
   $(2+x)(1+\frac{1}{2}x+\frac{3}{8}x^2)$.
4. M1 for calculating terms up to x^2:
   $2(1+\frac{1}{2}x+\frac{3}{8}x^2) + x(1+\frac{1}{2}x)$.
5. A1 for the final simplified expression: $2 + 2x + \frac{5}{4}x^2$.

Question 10 — 5 marks

1. M1 for stating
   $\frac{3}{(1-x)(1+2x)} = (1-x)^{-1} + 2(1+2x)^{-1}$.
2. M1 for expanding $(1-x)^{-1} \approx 1 + x + x^2$.
3. M1 for expanding
   $2(1+2x)^{-1} \approx 2(1-2x+4x^2) = 2 - 4x + 8x^2$.
4. M1 for adding the two expansions together.
5. A1 for the correctly simplified expression:
   $3 - 3x + 9x^2$.

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