P3 Trigonometric Functions: exam questions

10 questions · 34 marks · full mark schemes

Name _____ Date _____ Score / 34

- sec, cosec and cot; arcsin, arccos, arctan and their domains/ranges; $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p3/trigonometric-functions-questions

1. **2 marks · warm-up**

Find the exact value of $\arcsin\left(-\frac{1}{2}\right)$ in radians.

2. **2 marks · standard**

Solve the equation $\arcsin(2x) = \frac{\pi}{6}$.

3. **3 marks · standard**

Given that $\sin\theta = \frac{3}{5}$ and θ is an obtuse angle, find the exact value of $\sec\theta$.

4. 3 marks · standard

Solve the equation $\sec x = 2$ for $0° \leq x \leq 360°$.

5. 3 marks · standard

Sketch the graph of $y = \arcsin x$. Clearly state its domain and range.

6. 3 marks · standard

Solve $\sec^2 x = 4$ for $0 \leq x \leq \pi$.

7. 4 marks · harder

Prove the identity $\frac{1+\cot x}{1+\tan x} \equiv \cot x$.

8. 4 marks · harder

Solve the equation $3\sec^2\theta - 4 = 0$ for $0 \leq \theta \leq \pi$.

9. 4 marks · harder

Find the exact value of $\sec\left(\arcsin\frac{5}{13}\right)$.

10. 6 marks · harder

Solve $2\sec^2 x - \tan x = 5$ for $0 \leq x \leq 2\pi$. Give your answers to 3 significant figures.

Answers and worked solutions

Question 1 — 2 marks

1. M1 for recalling that the range of $\arcsin x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
2. A1 for stating the correct exact negative principal value $-\frac{\pi}{6}$.

Question 2 — 2 marks

1. M1 for taking the sine of both sides to get $2x = \sin(\frac{\pi}{6}) = 0.5$.
2. A1 for solving for x to get $x = 0.25$.

Question 3 — 3 marks

1. M1 for using the identity $\cos^2\theta = 1 - \sin^2\theta$ to find $\cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$.
2. M1 for deducing that since θ is obtuse, $\cos\theta$ is negative, so $\cos\theta = -\frac{4}{5}$.
3. A1 for the reciprocal $\sec\theta = -\frac{5}{4}$.

Question 4 — 3 marks

1. M1 for converting to cosine: $\cos x = \frac{1}{2}$.
2. M1 for finding the principal value $x = 60°$.
3. A1 for finding the second solution in the 4th quadrant: $x = 300°$.

Question 5 — 3 marks

1. M1 for sketching the standard curve of $y = \arcsin x$ passing through the origin with positive gradient.
2. A1 for stating the domain is $-1 \le x \le 1$.
3. A1 for stating the range is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Question 6 — 3 marks

1. M1 for rewriting as $\cos^2 x = \frac{1}{4}$.
2. M1 for taking the square root to find $\cos x = \pm 0.5$.
3. A1 for finding the two valid angles in the specified range: $x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$.

Question 7 — 4 marks

1. M1 for substituting $\tan x = \frac{1}{\cot x}$ into the denominator: $1 + \frac{1}{\cot x}$.
2. M1 for obtaining a common denominator in the denominator: $\frac{\cot x + 1}{\cot x}$.
3. M1 for writing the whole fraction as $(1 + \cot x) \div \frac{\cot x + 1}{\cot x}$.
4. A1 for multiplying by the reciprocal to show $(1 + \cot x) \times \frac{\cot x}{1 + \cot x} \equiv \cot x$.

Question 8 — 4 marks

1. M1 for rearranging to make $\sec^2\theta$ the subject: $\sec^2\theta = \frac{4}{3}$.
2. M1 for converting to cosine: $\cos^2\theta = \frac{3}{4}$.
3. M1 for taking the square root to find $\cos\theta = \pm\frac{\sqrt{3}}{2}$.
4. A1 for finding the valid solutions in the range: $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

Question 9 — 4 marks

1. M1 for letting $\alpha = \arcsin\frac{5}{13}$ such that $\sin\alpha = \frac{5}{13}$.
2. M1 for using a Pythagorean triple $(5, 12, 13)$ or identity to find the adjacent side is 12.
3. M1 for deducing $\cos\alpha = \frac{12}{13}$.
4. A1 for finding the reciprocal $\sec\alpha = \frac{13}{12}$.

Question 10 — 6 marks

1. M1 for substituting $\sec^2 x = 1 + \tan^2 x$ to get $2(1 + \tan^2 x) - \tan x - 5 = 0$.
2. M1 for forming the quadratic $2\tan^2 x - \tan x - 3 = 0$.
3. M1 for factorising the quadratic to $(2\tan x - 3)(\tan x + 1) = 0$.
4. A1 for solving $\tan x = 1.5$ to get $x = 0.983$ and 4.12.
5. A1 for solving $\tan x = -1$ to get exact or decimal equivalents $x = \frac{3\pi}{4} \approx 2.36$ and $x = \frac{7\pi}{4} \approx 5.50$.
6. A1 for stating all four answers clearly.

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