P3 Trigonometric Addition Formulae: exam questions

10 questions · 37 marks · full mark schemes

Name _____ Date _____ Score / 37

- Addition and double-angle formulae; a cos θ + b sin θ in the form R cos(θ ± α) or R sin(θ ± α).
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p3/trig-addition-formulae-questions

1. **2 marks · warm-up**

Simplify the expression $\cos\left(\theta + \frac{\pi}{2}\right)$ using the addition formulae.

2. **3 marks · warm-up**

Given that $\sin\theta = \frac{5}{13}$ and θ is an acute angle, find the exact value of $\sin 2\theta$.

3. **3 marks · standard**

Given that $\tan A = 3$ and $\tan B = \frac{1}{2}$, find the exact value of $\tan(A - B)$.

4. 3 marks · standard

Prove the identity $\frac{\sin(A-B)}{\cos A \cos B} \equiv \tan A - \tan B$.

5. 3 marks · standard

Prove that $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$.

6. 6 marks · standard

Solve the equation $\frac{\sqrt{3}}{2}\cos 2x = \sin x \cos x$ for $0 \leq x \leq \pi$. Give your answers as exact radian values.

7. 4 marks · harder

Given that $\cos A = -\frac{3}{5}$ and A is obtuse, find the exact value of $\cos\left(A + \frac{\pi}{3}\right)$.

8. 4 marks · harder

Express $\sin 3\theta$ entirely in terms of $\sin\theta$.

9. 4 marks · harder

Find the maximum value of $24\sin x + 7\cos x$ and the smallest positive value of x at which it occurs. Give your value of x to 1 decimal place.

10. 5 marks · harder

Solve $5\cos 2x + 7\sin 2x = 3$ for $0° \leq x \leq 180°$. Give your answers to 1 decimal place.

Answers and worked solutions

Question 1

2 marks

1. M1 for expanding to $\cos\theta\cos\frac{\pi}{2} - \sin\theta\sin\frac{\pi}{2}$.
2. A1 for substituting $\cos\frac{\pi}{2} = 0$ and $\sin\frac{\pi}{2} = 1$ to obtain $-\sin\theta$.

Question 2

3 marks

1. M1 for finding $\cos\theta = \frac{12}{13}$.
2. M1 for using the double-angle formula $\sin 2\theta = 2\sin\theta\cos\theta$.
3. A1 for calculating $2(\frac{5}{13})(\frac{12}{13}) = \frac{120}{169}$.

Question 3

3 marks

1. M1 for applying the formula $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$.
2. M1 for substituting the given values: $\frac{3-0.5}{1+3(0.5)}$.
3. A1 for simplifying to $\frac{2.5}{2.5} = 1$.

Question 4

3 marks

1. M1 for expanding the numerator: $\sin A\cos B - \cos A\sin B$.
2. M1 for separating the fraction into $\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}$.
3. A1 for cancelling the cosine terms to yield $\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B} \equiv \tan A - \tan B$.

Question 5

3 marks

1. M1 for factorising the difference of two squares: $(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$.
2. M1 for applying the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$.
3. A1 for stating that the remaining factor $\cos^2\theta - \sin^2\theta$ is exactly the double-angle identity for $\cos 2\theta$.

Question 6

6 marks

Answer: $x = \frac{\pi}{6}, \frac{2\pi}{3}$

Recognise that $\sin x\cos x = \frac{1}{2}\sin 2x$. **M1**

The equation becomes $\frac{\sqrt{3}}{2}\cos 2x = \frac{1}{2}\sin 2x$. **A1**

Multiply both sides by 2 and divide by $\cos 2x$:

$\sqrt{3} = \frac{\sin 2x}{\cos 2x} \implies \tan 2x = \sqrt{3}$. **M1**

Since $0 \le x \le \pi$, the domain for $2x$ is $0 \le 2x \le 2\pi$. **B1**

The reference angle for $\tan(2x) = \sqrt{3}$ is $\frac{\pi}{3}$.

In $0 \le 2x \le 2\pi$, $2x = \frac{\pi}{3}$ and $2x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$. **A1**

Divide by 2: $x = \frac{\pi}{6}$ and $x = \frac{2\pi}{3}$. **A1**

Question 7

4 marks

1. M1 for deducing that since A is obtuse, $\sin A$ must be positive, giving $\sin A = \frac{4}{5}$.
2. M1 for applying the expansion $\cos(A + \frac{\pi}{3}) = \cos A\cos\frac{\pi}{3} - \sin A\sin\frac{\pi}{3}$.
3. M1 for substituting the exact values: $(-\frac{3}{5})(\frac{1}{2}) - (\frac{4}{5})(\frac{\sqrt{3}}{2})$.
4. A1 for simplifying to $\frac{-3-4\sqrt{3}}{10}$.

Question 8

4 marks

1. M1 for writing $\sin 3\theta = \sin(2\theta+\theta)$ and expanding to $\sin 2\theta\cos\theta + \cos 2\theta\sin\theta$.
2. M1 for substituting the double-angle formulae $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$.
3. M1 for multiplying to get $2\sin\theta\cos^2\theta + \sin\theta(1 - 2\sin^2\theta)$ and replacing $\cos^2\theta$ with $1 - \sin^2\theta$.
4. A1 for fully expanding and simplifying the terms to obtain $3\sin\theta - 4\sin^3\theta$.

Question 9

4 marks

1. M1 for expressing $24\sin x + 7\cos x$ in the harmonic form $R\sin(x+\alpha)$, finding $R = \sqrt{24^2 + 7^2} = 25$.
2. M1 for finding $\tan\alpha = \frac{7}{24} \implies \alpha = 16.3°$, giving the expression $25\sin(x + 16.3°)$.
3. A1 for stating that the maximum value is 25.
4. A1 for solving $x + 16.3° = 90°$ to find the smallest positive $x = 73.7°$.

Question 10

5 marks

1. M1 for expressing $5\cos 2x + 7\sin 2x$ in the form $R\cos(2x - \alpha)$: finding $R = \sqrt{25+49} = \sqrt{74}$ and $\tan\alpha = \frac{7}{5} \implies \alpha = 54.46°$ (keep this unrounded).
2. M1 for rewriting as $\sqrt{74}\cos(2x - 54.46°) = 3 \implies \cos(2x - 54.46°) = \frac{3}{\sqrt{74}} \approx 0.3487$.
3. M1 for evaluating the principal angle $\arccos(0.3487) = 69.59°$.
4. A1 for the first solution in range: $2x - 54.46° = 69.59° \implies 2x = 124.05° \implies x = 62.0°$.
5. A1 for the second solution: $-69.59°$ is out of range, so use $360° - 69.59° = 290.41° \implies 2x = 344.87° \implies x = 172.4°$.

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