P3 Numerical Methods: exam questions

10 questions · 31 marks · full mark schemes

Name _____ Date _____ Score / 31

- Locating roots of f(x) = 0 by a change of sign; approximate solutions using iteration $x_{n+1} = g(x_n)$, including cobweb/staircase diagrams. (Newton–Raphson is Further Pure 1, not P3.)
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p3/numerical-methods-questions

1. **2 marks · warm-up**

A student uses an iterative formula to find a root of an equation. The successive values oscillate around the root, getting closer to it with each iteration. Describe the type of diagram that would visually represent this convergence.

2. **2 marks · warm-up**

A student uses an iterative formula to find a root of an equation. The successive values approach the root from one side only, consistently increasing or decreasing. Describe the type of diagram that would visually represent this convergence.

3. **3 marks · warm-up**

Show that the root of $f(x) = e^x - x - 3 = 0$ lies in the interval $[1.5, 1.6]$.

4. 3 marks · standard

Explain why attempting to find a root of $f(x) = x^2$ by looking for a sign change between $x = -1$ and $x = 1$ will fail, even though there is a root at $x = 0$.

5. 3 marks · standard

Show that the equation $2x^3 - 5x^2 + 3 = 0$ can be rearranged into the iterative formula $x_{n+1} = \frac{2x_n^3+3}{5x_n}$.

6. 4 marks · standard

A root α of the equation $x^3 - 4x + 1 = 0$ lies in the interval $[0.2, 0.3]$. By choosing a suitable interval, verify that $\alpha = 0.25$ correct to 2 decimal places.

7. 4 marks · standard

The equation $x^3 - 3x^2 + 5 = 0$ can be rearranged into $x = \sqrt{\frac{x^3+5}{3}}$. If this iterative formula is used with $x_0 = 2$, calculate x_1, x_2, and x_3 to 3 decimal places.

8. 5 marks · standard

Show that the equation $x = \sqrt[3]{2x+3}$ has a root in $[1, 2]$, and use the iteration $x_{n+1} = \sqrt[3]{2x_n+3}$ with $x_0 = 1.5$ to find x_1, x_2, x_3 to 4 d.p.

9. 2 marks · harder

Explain the significance of the gradient of the curve $y = g(x)$ near a root α when using the iterative formula $x_{n+1} = g(x_n)$.

10. 3 marks · harder

The function $f(x) = \frac{x^2-4x+3}{x-2}$ has roots at $x = 1$ and $x = 3$. A student notes a sign change between $x = 1.5$ and $x = 2.5$. Explain why this does not imply a root in the interval $(1.5, 2.5)$.

Answers and worked solutions

Question 1 — 2 marks

1. M1 for recognising the pattern of alternating above and below the root during convergence.
2. A1 for correctly identifying this as a 'cobweb diagram'.

Question 2 — 2 marks

1. M1 for recognising the pattern of approaching the root from a single direction.
2. A1 for correctly identifying this as a 'staircase diagram'.

Question 3 — 3 marks

1. M1 for evaluating $f(1.5)$ and $f(1.6)$.
2. A1 for finding $f(1.5) = e^{1.5} - 4.5 \approx -0.018$ and $f(1.6) = e^{1.6} - 4.6 \approx 0.353$.
3. A1 for stating that the sign change in a continuous function confirms a root lies in the interval.

Question 4 — 3 marks

1. M1 for calculating $f(-1) = 1$ and $f(1) = 1$.
2. A1 for observing that there is no change of sign between the two values evaluated.
3. A1 for explaining that $x = 0$ is a turning point (local minimum) that touches the x-axis without crossing it, so the function does not change sign.

Question 5 — 3 marks

1. M1 for rewriting the equation as $5x^2 = 2x^3 + 3$.
2. M1 for dividing by $5x$ to isolate one x on the left-hand side.
3. A1 for reaching the exact required form $x_{n+1} = \frac{2x_n^3+3}{5x_n}$.

Question 6 — 4 marks

1. M1 for identifying the correct test bounds: 0.245 and 0.255.
2. M1 for evaluating $f(0.245) = (0.245)^3 - 4(0.245) + 1 \approx 0.0347$.
3. M1 for evaluating $f(0.255) = (0.255)^3 - 4(0.255) + 1 \approx -0.0034$.
4. A1 for concluding that the change of sign between 0.245 and 0.255 confirms the root is exactly 0.25 when rounded to 2 decimal places.

Question 7 — 4 marks

1. M1 for substituting $x_0 = 2$ to find $x_1 = \sqrt{(8+5)/3}$.
2. A1 for $x_1 = 2.082$.
3. A1 for evaluating $x_2 = \sqrt{((2.0816\ldots)^3 + 5)/3} = 2.162$.
4. A1 for evaluating $x_3 = \sqrt{((2.1618\ldots)^3 + 5)/3} = 2.244$.

Question 8 — 5 marks

Answer: $x_1 = 1.8171$, $x_2 = 1.8790$, $x_3 = 1.8906$

Let $g(x) = x^3 - 2x - 3$. $g(1) = -4 < 0$;
$g(2) = 1 > 0 \Rightarrow$ root in $(1, 2)$ **M1 A1**.
$x_1 = \sqrt[3]{6} = 1.8171$ **M1 A1**.
$x_2 = \sqrt[3]{6.6342} = 1.8790$;
$x_3 = \sqrt[3]{6.7580} = 1.8906$ **A1**.

Question 9 — 2 marks

1. M1 for identifying that the gradient determines whether the iteration converges or diverges.
2. A1 for stating that for the iteration to converge to the root α, the condition $|g'(\alpha)| < 1$ must be satisfied.

Question 10 — 3 marks

1. M1 for evaluating $f(1.5) = 1.5$ and $f(2.5) = -1.5$, identifying the positive-to-negative sign change.
2. M1 for examining the denominator and noting that $f(x)$ is undefined at $x = 2$.
3. A1 for concluding that the function has a vertical asymptote at $x = 2$, meaning it is not continuous over the interval $(1.5, 2.5)$, so the sign change rule cannot be applied to deduce a root.

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