P3 Further Integration: exam questions

10 questions · 28 marks · full mark schemes

Name _____ Date _____ Score / 28

- Integrating e^x, 1/x, sin x and cos x, including f(ax + b) forms (substitution and parts are P4).
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p3/further-integration-questions

1. **2 marks · warm-up**

Find $\int \cos(3x)\,dx$.

2. **2 marks · warm-up**

Find $\int \frac{5}{x}\,dx$.

3. **2 marks · standard**

Find $\int \frac{\sec^2 x}{\tan x}\,dx$.

4. 3 marks · standard

Find $\int \tan^2 x \, dx$.

5. 3 marks · standard

Find $\int \sin x \cos^4 x \, dx$.

6. 4 marks · standard

Evaluate exactly $\int_0^1 \frac{4x}{2x^2+1} \, dx$.

7. 3 marks · harder

Find $\int \cos^2 x \, dx$.

8. 3 marks · harder

Find $\int \tan x \, dx$.

9. 3 marks · harder

Find $\int \frac{e^{2x}}{e^{2x}+1}\,dx$.

10. 3 marks · harder

Find $\int \frac{1+x}{x^2+2x-5}\,dx$.

Answers and worked solutions

Question 1 — 2 marks

1. M1 for applying the rule $\int \cos(kx)\,dx = \frac{1}{k}\sin(kx)$.
2. A1 for $\frac{1}{3}\sin(3x) + c$.

Question 2 — 2 marks

1. M1 for recognising the integral of $\frac{1}{x}$ is $\ln|x|$.
2. A1 for $5\ln|x| + c$.

Question 3 — 2 marks

1. M1 for recognising the numerator is the derivative of the denominator.
2. A1 for $\ln|\tan x| + c$.

Question 4 — 3 marks

1. M1 for using the trigonometric identity $\tan^2 x \equiv \sec^2 x - 1$.
2. M1 for integrating $\sec^2 x$ to $\tan x$ and -1 to $-x$.
3. A1 for $\tan x - x + c$.

Question 5 — 3 marks

1. M1 for recognising the form $\int kf'(x)[f(x)]^n\,dx$.
2. M1 for increasing the power of $\cos x$ to 5.
3. A1 for $-\frac{1}{5}\cos^5 x + c$.

Question 6 — 4 marks

1. M1 for recognising the numerator is the derivative of the denominator.
2. M1 for integrating to $[\ln(2x^2+1)]_0^1$.
3. M1 for substituting the limits: $\ln(3) - \ln(1)$.
4. A1 for $\ln 3$.

Question 7 — 3 marks

1. M1 for using the double angle identity $\cos^2 x \equiv \frac{1}{2}(1 + \cos 2x)$.
2. M1 for integrating to get $\frac{1}{2}(x + \frac{1}{2}\sin 2x)$.
3. A1 for $\frac{1}{2}x + \frac{1}{4}\sin 2x + c$.

Question 8 — 3 marks

1. M1 for rewriting $\tan x$ as $\frac{\sin x}{\cos x}$.
2. M1 for recognising this is of the form $-\int \frac{-\sin x}{\cos x}\,dx$.
3. A1 for $-\ln|\cos x| + c$ or $\ln|\sec x| + c$.

Question 9 — 3 marks

1. M1 for recognising the form $\int \frac{kf'(x)}{f(x)}\,dx$.
2. M1 for adjusting the constant by $\frac{1}{2}$.
3. A1 for $\frac{1}{2}\ln(e^{2x}+1) + c$.

Question 10 — 3 marks

1. M1 for differentiating the denominator to get $2x + 2 = 2(x+1)$.
2. M1 for recognising the form $\frac{1}{2}\int \frac{f'(x)}{f(x)}\,dx$.
3. A1 for $\frac{1}{2}\ln|x^2 + 2x - 5| + c$.

More further integration practice with instant feedback: alevelmathrevision.com/alevel-frontend/units/p3/further-integration-questions

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