P3 Further Differentiation: exam questions

10 questions · 30 marks · full mark schemes

Name _____ Date _____ Score / 30

- Chain, product and quotient rules; differentiating e^x, ln x, sin x, cos x, tan x (and sec, cosec, cot); dy/dx = 1/(dx/dy).
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p3/further-differentiation-questions

1. **2 marks · warm-up**

Differentiate $y = (3x^2 - x)^4$ with respect to x.

2. **2 marks · warm-up**

Differentiate $y = \sin(5x) - \cos(2x)$ with respect to x.

3. **2 marks · standard**

Find $\frac{dy}{dx}$ when $y = 2^{3x-1}$.

4. **2 marks · standard**

Given $y = \ln(3x + 1)$, find $\frac{dy}{dx}$.

5. 3 marks · standard

Use the product rule to differentiate $y = x^2e^{3x}$ with respect to x. Factorise your final answer completely.

6. 3 marks · standard

Given that $y = \frac{\sin x}{x}$, find $\frac{dy}{dx}$ using the quotient rule.

7. 4 marks · harder

Differentiate $y = \frac{e^{2x}}{x^2+1}$ with respect to x, giving your answer as a single algebraic fraction in its simplest form.

8. 4 marks · harder

Show that the derivative of $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is always positive for all real values of x.

9. 4 marks · harder

Find the exact gradient of the curve $y = \operatorname{cosec}(2x)$ at the point where $x = \frac{\pi}{6}$.

10. 4 marks · harder

Show that if $y = x^x$ then $\frac{dy}{dx} = x^x(1 + \ln x)$.

Answers and worked solutions

Question 1

2 marks

1. M1 for applying the chain rule, decreasing the power by 1 and multiplying by the derivative of the inside bracket: $4(3x^2 - x)^3 \times \ldots$
2. A1 for the fully correct derivative: $4(6x - 1)(3x^2 - x)^3$.

Question 2

2 marks

1. M1 for applying the standard rules for differentiating sine and cosine to give terms of the form $A\cos(5x)$ and $B\sin(2x)$.
2. A1 for the completely correct derivative: $5\cos(5x) + 2\sin(2x)$.

Question 3

2 marks

1. M1 for applying the rule for differentiating $a^{f(x)}$ to get a term involving $2^{3x-1}\ln 2$.
2. A1 for multiplying by the derivative of the exponent (3) to get $3 \times 2^{3x-1}\ln 2$.

Question 4

2 marks

Answer: $\dfrac{3}{3x+1}$

$\frac{dy}{dx} = \frac{k}{3x+1}$ **M1**

$= \frac{3}{3x+1}$ **A1**

Answer: $\dfrac{3}{3x+1}$

Question 5

3 marks

1. M1 for applying the product rule with $u = x^2$ and $v = e^{3x}$ to get an unsimplified expression like $2xe^{3x} + x^2(3e^{3x})$.
2. M1 for extracting the common factors x and e^{3x}.
3. A1 for the correctly factorised final answer: $xe^{3x}(3x + 2)$.

Question 6

3 marks

1. M1 for setting $u = \sin x$ and $v = x$, and finding $u' = \cos x$ and $v' = 1$.
2. M1 for substituting into the quotient rule formula: $\frac{vu' - uv'}{v^2}$.
3. A1 for the correctly simplified expression: $\frac{x\cos x - \sin x}{x^2}$.

Question 7

4 marks

1. M1 for correctly identifying $u = e^{2x}$ and $v = x^2 + 1$, and their derivatives $2e^{2x}$ and $2x$.
2. M1 for substituting into the quotient rule: $\frac{(x^2+1)2e^{2x} - e^{2x}(2x)}{(x^2+1)^2}$.
3. M1 for factorising out $2e^{2x}$ in the numerator.
4. A1 for the correct simplified expression: $\frac{2e^{2x}(x^2 - x + 1)}{(x^2+1)^2}$.

Question 8

4 marks

1. M1 for correctly identifying u and v and their derivatives, and applying the quotient rule.
2. M1 for expanding the numerator: $(e^x + e^{-x})^2 - (e^x - e^{-x})^2$.
3. M1 for simplifying the numerator using difference of two squares or expansion to yield exactly 4.
4. A1 for stating $\frac{dy}{dx} = \frac{4}{(e^x + e^{-x})^2}$ and noting that since the numerator is positive and the denominator is a squared term (which is strictly positive), the derivative is always positive.

Question 9

4 marks

1. M1 for differentiating to get $\frac{dy}{dx} = -2\text{cosec}(2x)\cot(2x)$.
2. M1 for substituting $x = \frac{\pi}{6}$ into the derivative: $-2\text{cosec}(\frac{\pi}{3})\cot(\frac{\pi}{3})$.
3. M1 for substituting the exact trigonometric values: $\text{cosec}(\frac{\pi}{3}) = \frac{2}{\sqrt{3}}$ and $\cot(\frac{\pi}{3}) = \frac{1}{\sqrt{3}}$.
4. A1 for evaluating the final gradient as exactly $-\frac{4}{3}$.

Question 10

4 marks

Answer: Proven

$\ln y = x\ln x$ **M1**

$\frac{1}{y}\frac{dy}{dx}$ on the left **M1**

$= \ln x + x \cdot \frac{1}{x} = \ln x + 1$ **A1**

$\frac{dy}{dx} = y(1 + \ln x) = x^x(1 + \ln x)$ * **A1**

Answer: Proof (given result)

More further differentiation practice with instant feedback: alevelmathrevision.com/alevel-frontend/units/p3/further-differentiation-questions

Our own original questions, written and checked by A Level Math Revision. Independent, not affiliated with Pearson Edexcel.