P3 Functions: exam questions

10 questions · 30 marks · full mark schemes

Name _____ Date _____ Score / 30

- Domain, range, composite and inverse functions; the modulus function y = |f(x)| and y = f(|x|); combinations of transformations.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p3/functions-questions

1. **2 marks · warm-up**

The function $f(x)$ is defined by $f(x) = x^2 - 4x + 7$ for $x \geq 2$. Find the range of $f(x)$.

2. **3 marks · warm-up**

Solve the equation $|3x - 2| = 8$.

3. **2 marks · standard**

The function f is defined by $f(x) = \sqrt{x - 4}$ for $x \geq 4$. State the domain and range of the inverse function $f^{-1}(x)$.

4. 2 marks · standard

Find the exact range of the function $f(x) = 5 - 2|x + 1|$.

5. 3 marks · standard

Find the set of values of x for which $|4x - 3| > 5$.

6. 3 marks · standard

Given the function $f(x) = 2x - 4$, sketch the graph of $y = f(|x|)$. State the coordinates of the turning point and any x-intercepts.

7. 3 marks · harder

A function f satisfies $f(x + 2) = 3f(x)$ and $f(0) = 1$. Find $f(6)$.

8. 4 marks · harder

Solve the equation $|x - 2| = |3x + 1|$.

9. **4 marks · harder**

Given that $f(x) = x^2 - 2$ for $x \geq 0$, solve the equation $f(x) = f^{-1}(x)$.

10. **4 marks · harder**

Find the coordinates of the points of intersection of the graphs $y = 3|x|$ and $y = 10 - 2x$.

Answers and worked solutions

Question 1

2 marks

1. M1 for completing the square or differentiating to find the minimum:
   $f(x) = (x-2)^2 - 4 + 7 = (x-2)^2 + 3$.
2. A1 for deducing that since $(x-2)^2 \geq 0$, the minimum value is 3, so the correct range is $f(x) \geq 3$.

Question 2

3 marks

1. M1 for removing the modulus to give two linear equations: $3x - 2 = 8$ and $3x - 2 = -8$.
2. M1 for solving at least one equation correctly.
3. A1 for stating the final two solutions $x = \frac{10}{3}$ and $x = -2$.

Question 3

2 marks

1. M1 for identifying the range of the original function $f(x) \geq 0$ which maps to the domain of the inverse.
2. A1 for stating the domain of $f^{-1}(x)$ is $x \geq 0$ and the range is $f^{-1}(x) \geq 4$.

Question 4

2 marks

1. M1 for recognising that $|x+1| \geq 0$ for all real x, which means $2|x+1|$ subtracts a non-negative amount from 5.
2. A1 for deducing the maximum value is $5 - 0 = 5$, giving the correct range $f(x) \leq 5$.

Question 5

3 marks

1. M1 for setting up the two linear inequalities: $4x - 3 > 5$ and $4x - 3 < -5$.
2. M1 for solving both inequalities to get $4x > 8$ and $4x < -2$.
3. A1 for stating the final solution sets $x > 2$ or $x < -0.5$.

Question 6

3 marks

1. M1 for sketching $y = 2x - 4$ for $x \geq 0$, which is a straight line starting at $(0, -4)$ and passing through $(2, 0)$.
2. M1 for reflecting this part in the y-axis to create a V-shape.
3. A1 for clearly indicating the vertex at $(0, -4)$ and the two x-intercepts at $x = 2$ and $x = -2$.

Question 7

3 marks

Answer: 27

$f(2) = 3f(0) = 3$ **M1**
$f(4) = 3f(2) = 9$ **A1**
$f(6) = 3f(4) = 27$ **A1**

Answer: 27

Question 8

4 marks

1. M1 for squaring both sides: $(x-2)^2 = (3x+1)^2$. (Alternatively, setting $x - 2 = \pm(3x+1)$).
2. M1 for expanding correctly: $x^2 - 4x + 4 = 9x^2 + 6x + 1$.
3. M1 for rearranging into a standard quadratic: $8x^2 + 10x - 3 = 0$ and attempting to factorise $(4x-1)(2x+3) = 0$.
4. A1 for stating the correct exact solutions $x = 0.25$ and $x = -1.5$.

Question 9

4 marks

1. M1 for using the property that $f(x) = f^{-1}(x)$ intersects on the line $y = x$, thus equating $f(x)$ to x: $x^2 - 2 = x$.
2. M1 for rearranging into a standard quadratic equation: $x^2 - x - 2 = 0$.
3. M1 for factorising or solving: $(x-2)(x+1) = 0$.
4. A1 for selecting the valid root $x = 2$ and explicitly rejecting $x = -1$ due to the domain condition $x \geq 0$.

Question 10

4 marks

1. M1 for solving the positive case where $x \geq 0$: $3x = 10 - 2x \implies 5x = 10$.
2. A1 for finding the intersection point $x = 2$ and $y = 6$ i.e. $(2, 6)$.
3. M1 for solving the negative case where $x < 0$: $-3x = 10 - 2x \implies -x = 10$.
4. A1 for finding the intersection point $x = -10$ and $y = 30$ i.e. $(-10, 30)$.

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