P3 Exponentials and Logarithms: exam questions

10 questions · 37 marks · full mark schemes

Name _____ Date _____ Score / 37

- e^x and ln x as inverse functions; exponential growth/decay; log-linear graphs for $y = ax^n$ and $y = kb^x$.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p3/exponentials-logarithms-questions

1. **3 marks · warm-up**

Differentiate $y = 3e^{4x} - 2e^{-x}$ with respect to x.

2. **3 marks · warm-up**

The population of a colony of bacteria, P, after t hours is modelled by the equation $P = 200e^{0.15t}$. Find the initial population and the population after 4 hours, giving your answer to the nearest whole number.

3. **3 marks · standard**

Given that $y = \ln(4x^3)$, express y in the form $A \ln x + B$, where A and B are constants to be found.

4. 4 marks · standard

The mass, M grams, of a radioactive substance at time t days is given by $M = 50e^{-kt}$, where k is a positive constant. Given that the mass decreases to 25 grams after 14 days, find the exact value of k.

5. 4 marks · standard

Solve $e^{2x} - e^{x} - 6 = 0$.

6. 4 marks · standard

The amount of a drug in a patient's bloodstream, D mg, t hours after it is administered is modelled by $D = 15e^{-0.1t}$. Determine the time taken for the amount of drug to decrease to 3 mg. Give your answer to 1 decimal place.

7. 4 marks · harder

Find the exact solution to the equation $5^{x-1} = 2^{x}$.

8. 4 marks · harder

Find the exact equation of the tangent to the curve $y = 2e^x + 1$ at the point $(0, 3)$.

9. 4 marks · harder

Find the equation of the normal to the curve $y = e^{3x}$ at the point where $x = 0$. Give your answer in the form $ax + by + c = 0$, where a, b, c are integers.

10. 4 marks · harder

The temperature, $T°\text{C}$, of a cup of tea t minutes after it is made is modelled by $T = 20 + 70e^{-0.08t}$. Find the initial temperature of the tea and the rate at which the temperature is decreasing when $t = 5$.

Answers and worked solutions

Question 1

3 marks

1. M1 for applying the rule $\frac{d}{dx}(e^{kx}) = ke^{kx}$ to at least one term.
2. A1 for the first term differentiating correctly to $12e^{4x}$.
3. A1 for the second term differentiating correctly to $+2e^{-x}$, giving $\frac{dy}{dx} = 12e^{4x} + 2e^{-x}$.

Question 2

3 marks

1. B1 for stating the initial population is 200 (by substituting $t = 0$).
2. M1 for substituting $t = 4$ into the model to get $P = 200e^{0.6}$.
3. A1 for evaluating the population to 364 bacteria.

Question 3

3 marks

1. M1 for applying the multiplication law of logarithms: $\ln(4x^3) = \ln 4 + \ln(x^3)$.
2. M1 for applying the power law of logarithms to the second term: $\ln 4 + 3\ln x$.
3. A1 for correctly stating $A = 3$ and $B = \ln 4$.

Question 4

4 marks

1. M1 for substituting the given values $M = 25$ when $t = 14$: $25 = 50e^{-14k}$.
2. M1 for dividing by 50 to isolate the exponential: $e^{-14k} = 0.5$.
3. M1 for converting out of exponential form to natural logarithm form: $-14k = \ln 0.5$ (or $\ln \frac{1}{2}$).
4. A1 for stating the exact value of k as $-\frac{\ln 0.5}{14}$ or $\frac{\ln 2}{14}$.

Question 5

4 marks

1. M1 for recognising the quadratic form in e^x and factorising: $(e^x - 3)(e^x + 2) = 0$.
2. M1 for finding the potential solutions $e^x = 3$ and $e^x = -2$.
3. M1 for explaining that $e^x = -2$ must be rejected because $e^x > 0$ for all real x.
4. A1 for the exact and only valid solution $x = \ln 3$.

Question 6

4 marks

1. M1 for substituting $D = 3$ into the equation: $3 = 15e^{-0.1t}$.
2. M1 for dividing by 15 to isolate the exponential part: $e^{-0.1t} = 0.2$.
3. M1 for converting to logarithmic form: $-0.1t = \ln 0.2$.
4. A1 for calculating $t = -10\ln 0.2$ (or $10\ln 5$) ≈ 16.1 hours.

Question 7

4 marks

1. M1 for taking the natural logarithm of both sides: $(x-1)\ln 5 = x\ln 2$.
2. M1 for expanding the LHS: $x\ln 5 - \ln 5 = x\ln 2$.
3. M1 for collecting x terms on one side: $x(\ln 5 - \ln 2) = \ln 5$.
4. A1 for the final exact answer $x = \frac{\ln 5}{\ln 5 - \ln 2}$ or equivalent.

Question 8

4 marks

1. M1 for differentiating the function to find $\frac{dy}{dx} = 2e^x$.
2. M1 for evaluating the gradient at $x = 0$: $m = 2e^0 = 2$.
3. M1 for using the straight line equation format $y - y_1 = m(x - x_1)$ with the point $(0, 3)$ and $m = 2$.
4. A1 for the correct equation $y = 2x + 3$.

Question 9

4 marks

1. M1 for finding the corresponding y-coordinate at $x = 0$, which is $y = e^0 = 1$.
2. M1 for differentiating to $\frac{dy}{dx} = 3e^{3x}$ and finding the gradient of the tangent $m_t = 3$.
3. M1 for finding the gradient of the normal $m_n = -\frac{1}{3}$ and forming the equation $y - 1 = -\frac{1}{3}(x - 0)$.
4. A1 for rearranging to the required form: $x + 3y - 3 = 0$.

Question 10

4 marks

1. B1 for substituting $t = 0$ to find the initial temperature $T = 20 + 70 = 90°\text{C}$.
2. M1 for differentiating with respect to t to find the rate of change: $\frac{dT}{dt} = -5.6e^{-0.08t}$.
3. M1 for substituting $t = 5$ into the derivative: $\frac{dT}{dt} = -5.6e^{-0.4}$.
4. A1 for calculating and concluding the rate of decrease is $3.75°\text{C}$ per minute (allow -3.75).

More exponentials and logarithms practice with instant feedback: alevelmathrevision.com/alevel-frontend/units/p3/exponentials-logarithms-questions

Our own original questions, written and checked by A Level Math Revision. Independent, not affiliated with Pearson Edexcel.