P3 Algebraic Methods: exam questions

10 questions · 34 marks · full mark schemes

Name _____ Date _____ Score / 34

- Simplifying rational expressions (factorising, cancelling, algebraic division).
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p3/algebraic-methods-questions

1. **3 marks · warm-up**

Express $\frac{x^2+5x+6}{x^2-4} \times \frac{x-2}{x+3}$ as a single fraction in its simplest form.

2. **3 marks · warm-up**

Express $\frac{2}{x+1} + \frac{3}{x+2}$ as a single fraction in its simplest form.

3. **3 marks · standard**

Simplify $\frac{x^3-x^2}{x^2-1} \times \frac{x+1}{x^2}$ fully.

4. 3 marks · standard

Express $\frac{5}{x-3} - \frac{2}{x+4}$ as a single fraction in its simplest form.

5. 3 marks · standard

Express $\frac{2x+1}{x^2+4x+4} + \frac{3}{x+2}$ as a single fraction in its simplest form.

6. 3 marks · standard

By using algebraic long division, express $\frac{x^3+2x^2-x-2}{x-1}$ as a polynomial in x.

7. 4 marks · harder

Simplify $\frac{3}{x^2+x} - \frac{2}{x^2-1}$ fully.

8. 4 marks · harder

Express $\frac{x^3-x^2+2x-5}{x^2-3}$ in the form $Ax+B+\frac{Cx+D}{x^2-3}$.

9. 4 marks · harder

Use algebraic division to express $\frac{4x^3+2x^2-3x+1}{2x^2-x+1}$ in the form $Ax+B+\frac{Cx+D}{2x^2-x+1}$.

10. 4 marks · harder

Express $\frac{2x^3+5x^2-4}{x+3}$ in the form $ax^2+bx+c+\frac{d}{x+3}$, where a, b, c and d are constants to be found.

Answers and worked solutions

Question 1

3 marks

1. M1 for factorising the quadratic numerator to $(x+2)(x+3)$ and the difference of squares to $(x-2)(x+2)$.
2. M1 for writing out the full product and cancelling $(x+3)$, $(x-2)$, and $(x+2)$.
3. A1 for concluding that the expression simplifies fully to exactly 1.

Question 2

3 marks

1. M1 for identifying the common denominator $(x+1)(x+2)$.
2. M1 for forming the combined numerator: $2(x+2)+3(x+1)$.
3. A1 for expanding and simplifying the numerator to obtain $\frac{5x+7}{(x+1)(x+2)}$.

Question 3

3 marks

1. M1 for factorising x^3-x^2 to $x^2(x-1)$.
2. M1 for factorising x^2-1 to $(x-1)(x+1)$.
3. A1 for cancelling x^2, $(x-1)$, and $(x+1)$ to leave exactly 1.

Question 4

3 marks

1. M1 for finding the common denominator $(x-3)(x+4)$.
2. M1 for setting up the numerator taking care with signs: $5(x+4)-2(x-3)$.
3. A1 for expanding accurately to $5x+20-2x+6$ and simplifying to $\frac{3x+26}{(x-3)(x+4)}$.

Question 5

3 marks

1. M1 for recognising that $x^2+4x+4=(x+2)^2$.
2. M1 for creating the common denominator $(x+2)^2$ and setting up the numerator: $(2x+1)+3(x+2)$.
3. A1 for simplifying the numerator to get $\frac{5x+7}{(x+2)^2}$.

Question 6

3 marks

1. M1 for setting up long division and finding the first term x^2.
2. M1 for continuing the division to find the $+3x$ term.
3. A1 for completing the division to obtain the exact quotient x^2+3x+2 with no remainder.

Question 7

4 marks

1. M1 for factorising denominators: $x(x+1)$ and $(x-1)(x+1)$.
2. M1 for establishing the lowest common denominator $x(x-1)(x+1)$.
3. M1 for finding the new numerators: $3(x-1)-2x$.
4. A1 for the correctly simplified fraction $\frac{x-3}{x(x^2-1)}$ or $\frac{x-3}{x(x-1)(x+1)}$.

Question 8

4 marks

1. M1 for starting the algebraic division and finding $Ax=x$.
2. M1 for calculating the next quotient term $B=-1$.
3. M1 for calculating the linear remainder after subtracting $(-x^2+3)$ from $(-x^2+5x-5)$.
4. A1 for completing the expression correctly as $x-1+\frac{5x-8}{x^2-3}$.

Question 9

4 marks

1. M1 for obtaining the first term in the quotient: $2x$.
2. M1 for subtracting $2x(2x^2-x+1)=4x^3-2x^2+2x$ to get a new polynomial $4x^2-5x+1$, and finding the next quotient term $+2$.
3. M1 for finding the remainder by subtracting $2(2x^2-x+1)=4x^2-2x+2$.
4. A1 for the correct final answer $2x+2+\frac{-3x-1}{2x^2-x+1}$.

Question 10

4 marks

1. M1 for setting up the division and finding $a=2$.
2. M1 for continuing the division to get $-x^2-4$, yielding $b=-1$.
3. M1 for finding the constant term $c=3$ and calculating the remainder.
4. A1 for the completely correct expression $2x^2-x+3-\frac{13}{x+3}$.

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