EDEXCEL INTERNATIONAL A LEVEL · P2 PURE MATHEMATICS 2

P2 Sequences and Series: exam questions

10 questions · 35 marks · full mark schemes

Name _____ Date _____ Score / 35

- Sequences by nth term and recurrence (increasing, decreasing, periodic); arithmetic and geometric series, sum to n terms, sum to infinity (|r| < 1), Σ notation.
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p2/sequences-series-questions

1. **2 marks · warm-up**

Find the sum of the first 25 terms of the arithmetic series $10 + 13 + 16 + \ldots$

2. **2 marks · warm-up**

Find the sum of the first 10 terms of the geometric series $3 + 12 + 48 + 192 + \ldots$

3. **3 marks · standard**

The 5th term of an arithmetic sequence is 22 and the 10th term is 47. Find the first term, a, and the common difference, d.

4. **3 marks · standard**

The 3rd term of a geometric sequence is 36 and the 6th term is 972. Find the first term and the common ratio.

5. **4 marks · standard**

Find the least value of n such that the sum of the geometric series $2 + 6 + 18 + \ldots$ to n terms exceeds 500,000.

6. **4 marks · standard**

An engineer's starting salary is £30,000. She receives an annual raise of £1,200 every year thereafter. Calculate the total amount she will have earned after exactly 15 years.

7. **4 marks · harder**

The first three terms of an arithmetic sequence are $2k + 1$, $4k - 3$, and $5k + 2$. Find the value of the constant k and the 10th term of the sequence.

8. 4 marks · harder

Prove that the sum of the first n terms of an arithmetic series is $S_n = \frac{n}{2}(2a + (n-1)d)$.

9. 4 marks · harder

Given that $\sum_{r=1}^{k}(2r + 5) = 280$, find the value of the positive integer k.

10. 5 marks · harder

A sequence is defined by $u_{n+2} = 4u_{n+1} - 3u_n$, with $u_1 = 1$ and $u_2 = 3$. Find an expression for u_n in terms of n, given that it takes the form $u_n = A(3^{n-1}) + B$, where A and B are constants.

Answers and worked solutions

Question 1

2 marks

1. M1 for substituting $a = 10, d = 3, n = 25$ into the sum formula: $S_{25} = \frac{25}{2}(2(10) + 24(3))$.
2. A1 for correctly evaluating $12.5 \times 92 = 1150$.

Question 2

2 marks

1. M1 for substituting $a = 3, r = 4, n = 10$ into the geometric sum formula: $S_{10} = \frac{3(4^{10}-1)}{4-1}$.
2. A1 for correctly evaluating $1\,048\,575$.

Question 3

3 marks

1. M1 for setting up two simultaneous equations: $a + 4d = 22$ and $a + 9d = 47$.
2. M1 for subtracting the equations to solve for d: $5d = 25 \implies d = 5$.
3. A1 for substituting back to find a: $a + 20 = 22 \implies a = 2$.

Question 4

3 marks

1. M1 for setting up the equations $ar^2 = 36$ and $ar^5 = 972$.
2. M1 for dividing the equations to find $r^3 = 27 \implies r = 3$.
3. A1 for substituting back to find $a = 4$.

Question 5

4 marks

1. M1 for setting up the inequality: $\frac{2(3^n-1)}{3-1} > 500000$.
2. M1 for simplifying to $3^n > 500001$.
3. M1 for taking logarithms to solve for n: $n > \frac{\log 500001}{\log 3} \approx 11.9$.
4. A1 for stating the least integer value $n = 12$.

Question 6

4 marks

1. M1 for recognising this as the sum of an arithmetic series where $a = 30000$ and $d = 1200$.
2. M1 for writing $S_{15} = \frac{15}{2}(2(30000) + 14(1200))$.
3. M1 for evaluating the expression inside the bracket: $60000 + 16800 = 76800$.
4. A1 for concluding the total is £576,000.

Question 7

4 marks

1. M1 for using the property of common differences: $(4k - 3) - (2k + 1) = (5k + 2) - (4k - 3)$.
2. M1 for simplifying to a linear equation in k: $2k - 4 = k + 5$.
3. A1 for finding $k = 9$.
4. A1 for finding the first term $a = 19$, $d = 14$, and the 10th term $u_{10} = 19 + 9(14) = 145$.

Question 8

4 marks

1. M1 for writing out the series: $S_n = a + (a+d) + \ldots + (a+(n-2)d) + (a+(n-1)d)$.
2. M1 for writing the series out in reverse order underneath.
3. M1 for adding the two series together to get $2S_n = n \times (2a + (n-1)d)$.
4. A1 for correctly concluding $S_n = \frac{n}{2}(2a + (n-1)d)$.

Question 9

4 marks

1. M1 for identifying an arithmetic series with $a = 7$ and $d = 2$.
2. M1 for setting up the sum formula: $\frac{k}{2}(2(7) + (k-1)2) = 280$.
3. M1 for simplifying to form a quadratic: $k^2 + 6k - 280 = 0$.
4. A1 for solving $(k + 20)(k - 14) = 0$ and selecting $k = 14$.

Question 10

5 marks

1. M1 for substituting $n = 1$ into the given form to obtain $A(3^0) + B = 1 \implies A + B = 1$.
2. M1 for substituting $n = 2$ into the given form to obtain $A(3^1) + B = 3 \implies 3A + B = 3$.
3. M1 for solving the simultaneous equations to find $2A = 2 \implies A = 1$.
4. M1 for substituting A back to find B: $1 + B = 1 \implies B = 0$.
5. A1 for writing the final general expression $u_n = 3^{n-1}$.

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