P2 Proof: exam questions

5 questions · 17 marks · full mark schemes

Name _____ Date _____ Score / 17

- Proof by deduction, proof by exhaustion and disproof by counter-example, in number, algebra, inequalities and coordinate geometry (proof by contradiction is P4).
- Try every question before you look at the mark schemes. They start after the questions: M marks for method, A marks for accuracy, B marks for a correct result on its own.
- Online, with more practice: alevelmathrevision.com/alevel-frontend/units/p2/proof-questions

1. **2 marks · warm-up**

Show, by means of a counter-example, that the statement

"if $x^2 > 9$, then $x > 3$"

is not true for all real numbers x.

2. **3 marks · standard**

Prove that one more than the product of two consecutive even numbers is always a square number.

3. **3 marks · standard**

Show, by means of a counter-example, that the following statement is false.

"For all non-zero real numbers a and b, if $a > b$ then $\frac{1}{a} < \frac{1}{b}$."

4. **4 marks · harder**

The points $A(1,\ 2)$, $B(7,\ 4)$, $C(9,\ 10)$ and $D(3,\ 8)$ are the vertices of the quadrilateral $ABCD$.

Prove that $ABCD$ is a rhombus but not a square.

5. **5 marks · harder**

(a) Show that the statement

"$n^2 + 3n + 5$ is never a multiple of 3 for positive integers n"

is false. [2]

(b) Prove that $n^2 + 3n + 5$ is odd for every integer n. [3]

Answers and worked solutions

Question 1 — 2 marks

Answer: e.g. $x = -4$: $x^2 = 16 > 9$ but $x = -4 < 3$.

Chooses a value of x with $x < -3$, e.g. $x = -4$ **M1**

$(-4)^2 = 16 > 9$ but $-4 < 3$, so the statement is false **A1**

Note: any $x < -3$ earns both marks.

Question 2 — 3 marks

Answer:

$2n(2n + 2) + 1 = 4n^2 + 4n + 1 = (2n + 1)^2$.

Consecutive even numbers written as $2n$ and $2n + 2$, where n is an integer **B1**

$2n(2n + 2) + 1 = 4n^2 + 4n + 1$ **M1**

$= (2n + 1)^2$, the square of an integer, so always a square number **A1** (cso)

Question 3 — 3 marks

Answer: e.g. $a = 1$, $b = -1$: $a > b$ but $\frac{1}{a} = 1 > -1 = \frac{1}{b}$.

Chooses a and b of opposite signs with $a > b$, e.g. $a = 1$, $b = -1$ **M1**

$\dfrac{1}{a} = 1$ and $\dfrac{1}{b} = -1$ **A1**

$1 > -1$, so $\dfrac{1}{a} > \dfrac{1}{b}$ and the statement is false **A1**

Note: the statement is true when a and b have the same sign, so a correct counter-example must have $a > 0 > b$.

Question 4 — 4 marks

Answer: All sides $\sqrt{40}$; $m_{AB} \times m_{BC} = 1 \neq -1$, so no right angle.

Finds the lengths of the four sides, e.g. $AB = \sqrt{6^2 + 2^2}$ **M1**

$AB = BC = CD = DA = \sqrt{40}$, so $ABCD$ is a rhombus **A1**

Tests for a right angle, e.g. gradient $AB = \frac{1}{3}$, gradient $BC = 3$ **M1**

$\frac{1}{3} \times 3 = 1 \neq -1$, so AB is not perpendicular to BC: not a square **A1** (cso)

Question 5 — 5 marks

Answer: (a) $n = 1$: $9 = 3 \times 3$. (b) $n^2 + 3n = n(n + 3)$ is even (one of n, $n + 3$ is even), so $n^2 + 3n + 5$ is odd.

(a) $n = 1$: $1 + 3 + 5 = 9$ **M1**

$9 = 3 \times 3$ is a multiple of 3, so the statement is false **A1**

(b) $n^2 + 3n = n(n + 3)$ **M1**

If n is even, $n(n + 3)$ is even; if n is odd, $n + 3$ is even, so $n(n + 3)$ is even **A1**

So $n^2 + 3n + 5$ is even $+ 5$, which is odd, for every integer n **A1** (cso)

Alternative for (b): $n = 2k$ gives $4k^2 + 6k + 5 = 2(2k^2 + 3k + 2) + 1$; $n = 2k + 1$ gives $4k^2 + 10k + 9 = 2(2k^2 + 5k + 4) + 1$ (M1 one case, A1 both cases, A1 conclusion).

More proof practice with instant feedback: alevelmathrevision.com/alevel-frontend/units/p2/proof-questions
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